Term structure of risk in expected returns

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Introduction to the methodology: Campbell/Shiller decomp

- Campbell (1991) decomposition of shock to log asset return (here lowercase $r_t$)

$$r_{t,t+1} - E_t(r_{t,t+1}) = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^{j-1}(\Delta d_{t+j}) - (E_{t+1} - E_t) \sum_{j=2}^{\infty} \rho^{j-1} r_{t+j-1,t+j}$$

- Cash flow news

- Discount rate news

- Pure cash flow news $= \Delta$ has same Incremental effect on Expected multiperiod asset Returns

$$\mathcal{IER}(k = 1, \text{cash flow}) = E(r_{t,t+1} | \Delta) = \Delta$$

$$\mathcal{IER}(k > 1, \text{cash flow}) = E(r_{t,t+k} = \sum_{i=1}^{k} r_{t+i-1,t+i} | \Delta) = \Delta \forall k$$

- Pure discount rate news $= -\Delta$ has multiperiod effect that declines in $k$

$$\mathcal{IER}(k, \text{discount rate}) = E(r_{t,t+k} | \Delta) < \Delta, \ k > 1$$
Term structure of expectations as moments to match

An example: Bansal/Yaron (2004) case I: no s.v.

- A model in which variation in P/D is due to state variable driving expected future cash flows
  - In model, shock to P/D produces much more cash flow news than discount rate news (high EIS), close to a level shock to expected returns at all holding periods $k$

- Confront with the data, summarized by a VAR with returns and P/D: pos shock to P/D produces shocks to expected returns that decline substantially with holding period $k$
Extending this Campbell/Shiller logic

- Want similar term structure for models with state variables determining stochastic volatility, time-varying crash risk, . . .

Any feature not captured by the first-order approximation of Campbell/Shiller decomposition

- Given an asset-pricing model with a state vector $x_t$,

$$
\mathcal{IER}(\text{return horizon, } x \text{ shock at } t + 1) = \log E(\text{gross return}|\text{shock, } I_t) - \log E(\text{gross return}|I_t)
$$

- Model maps state vector to observables (like P/D, conditional return volatility)

VAR of returns, observables, plus mapping, determines empirical $\mathcal{IER}$; compare to $\mathcal{IER}$ implied by model’s restrictions
Interpreting the $I_\text{ER}$

- Campbell-Shiller intuition: term structure shape of $I_\text{ER}$ determined by relative importance of shocks to cash flow news, discount rate news

- My guess is same intuition applies here – what else do investors care about?
A toy model

- Periods 0, 1, 2
- Asset pays $e^{x_1 + x_2}$ at 2
- Uncertainty and timing of resolution
  - $x_1 = \bar{x} + e_1$, $e_1$ revealed at 1, $e_1 \sim N(0, \bar{V})$
  - $x_2 = \bar{x} + e_2$, $e_2$ revealed at 2
  - $e_{2|1} \sim N(0, \sigma_{2|1}^2)$, $\sigma_{2|1}^2 = \bar{V} + \eta_1$, $\eta_1$ revealed at 1,
    $\eta_1 \sim N(0, \sigma_{\eta}^2)$
- Valuation
  - $P_1 = e^{-d_1} E_1(e^{x_1 + x_2})$, $d_1 = \bar{d} + ae_1 + b\eta$
  - $P_0 = e^{-d_0} E_0(P_1)$, $d_0$ is fixed
Incremental Expected Return in the toy model

\[ \mathcal{IER}(k, \text{shock}) \]

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Cash flow news
Incremental Expected Return in the toy model

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| \( \chi_1 \) | 1 \( \quad \frac{1}{2} \quad a \quad \frac{1}{2} \) | 2 \|\]
| \( \sigma^2_{2|1} \) | 1 \( \quad b \quad 1 \) |

Cash flow news

Discount rate news
The Bansal-Yaron figure

- **Shock to growth rate of consumption**
  - Inferred from shock to P/D; model says P/D is cash-flow news; data disagree

- **Shock to conditional variances**
  - Data say small positive effect on multiperiod returns — next slide

**Figure 2**
The red dashed lines correspond to the theoretical term structures of risk. The blue solid
IER as a diagnostic tool?

- Inconsistent “advice” across examined models
  - Bansal/Yaron: empirically, positive shocks to conditional vols correspond to positive return shocks at all horizons
  - Drechsler/Yaron: empirically, positive shocks to conditional vols, time-varying mean of conditional vols, and jump in vols all correspond to negative return shocks at all horizons
- Possible partial explanation: estimated models disagree about the main drivers of P/D
  - P/D tends to pick up otherwise unobserved state variables
    - Example: P/D falls and conditional volatilities do not change—must be drop in expected consumption growth in Bansal/Yaron and increase in expected volatility growth in Drechsler/Yaron
Final comments

Term structure of shocks to expected returns is a nice idea that will benefit from

- Tighter intuition; if not cash flow news versus discount rate news, what is it?

- Model-free, or less model-dependent, stylized facts about IER\textsubscript{s} for different types of shocks