Term structure of risk in expected returns

Discussion by Greg Duffee, Johns Hopkins

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Introduction to the methodology: Campbell/Shiller decomp

- Campbell (1991) decomposition of shock to log asset return (here lowercase r_t)

$$r_{t,t+1} - E_t(r_{t,t+1}) = \underbrace{(E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^{j-1}(\Delta d_{t+j})}_{\text{cash flow news}} - \underbrace{(E_{t+1} - E_t)\sum_{j=2}^{\infty}\rho^{j-1} r_{t+j-1,t+j}}_{\text{discount rate news}}$$

- Pure cash flow news $= \Delta$ has same $\mathcal{I}$ncremental effect on $\mathcal{E}$xpected multiperiod asset $\mathcal{R}$eturns

$$\mathcal{IER}(k = 1, \text{cash flow}) = E(r_{t,t+1}|\Delta) = \Delta$$

$$\mathcal{IER}(k > 1, \text{cash flow}) = E(r_{t,t+k} = \sum_{i=1}^{k} r_{t+i-1,t+i}|\Delta) = \Delta \; \forall k$$

- Pure discount rate news $= -\Delta$ has multiperiod effect that declines in k

$$\mathcal{IER}(k, \text{discount rate}) = E(r_{t,t+k}|\Delta) < \Delta, \;\; k > 1$$

Term structure of expectations as moments to match

An example: Bansal/Yaron (2004) case **I**: no s.v.

- A model in which variation in P/D is due to state variable driving expected future cash flows
  - In model, shock to P/D produces much more cash flow news than discount rate news (high EIS), close to a level shock to expected returns at all holding periods k
- Confront with the data, summarized by a VAR with returns and P/D: pos shock to P/D produces shocks to expected returns that decline substantially with holding period k

Extending this Campbell/Shiller logic

- Want similar term structure for models with state variables determining stochastic volatility, time-varying crash risk, . . .

  Any feature not captured by the first-order approximation of Campbell/Shiller decomposition

- Given an asset-pricing model with a state vector x_t,

$$\mathcal{IER}(\text{return horizon}, x \text{ shock at } t+1) = \log E(\text{gross return}|\text{shock}, \mathcal{I}_t) - \log E(\text{gross return}|\mathcal{I}_t)$$

- Model maps state vector to observables (like P/D, conditional return volatility)

  VAR of returns, observables, plus mapping, determines empirical $\mathcal{IER}$; compare to $\mathcal{IER}$ implied by model's restrictions

Interpreting the $\mathcal{IER}$

- Campbell-Shiller intuition: term structure shape of $\mathcal{IER}$ determined by relative importance of shocks to cash flow news, discount rate news
- My guess is same intuition applies here – what else do investors care about?

A toy model

- Periods 0, 1, 2
- Asset pays $e^{x_1+x_2}$ at 2
- Uncertainty and timing of resolution
  - $x_1 = \overline{x} + e_1, \quad e_1$ revealed at 1, $e_1 \sim N(0, \overline{V})$
  - $x_2 = \overline{x} + e_2, \quad e_2$ revealed at 2
  - $e_{2|1} \sim N(0, \sigma^2_{2|1}), \quad \sigma^2_{2|1} = \overline{V} + \eta_1, \quad \eta_1$ revealed at 1, $\eta_1 \sim N(0, \sigma^2_\eta)$
- Valuation
  - $P_1 = e^{-d_1} E_1(e^{x_1+x_2}), \quad d_1 = \overline{d} + ae_1 + b\eta$
  - $P_0 = e^{-d_0} E_0(P_1), \quad d_0$ is fixed

Incremental Expected Return in the toy model

$\mathcal{IER}(k, \text{shock})$

	Horizon, k	
Shock to:	1	2
x_1	$1 - a$	1
$\sigma^2_{2\vert 1}$	$\frac{1}{2} - b$	$\frac{1}{2}$

Incremental Expected Return in the toy model

$\mathcal{IER}(k, \text{shock})$

Shock to:	Horizon, k 1	2
x_1	$1 - a$	1
$\sigma^2_{2\|1}$	$\frac{1}{2} - b$	$\frac{1}{2}$

Cash flow news

Incremental Expected Return in the toy model

$\mathcal{IER}(k, \text{shock})$

	Horizon, k	
Shock to:	1	2
x_1	$1 - a$	1
$\sigma^2_{2\|1}$	$\frac{1}{2} - b$	$\frac{1}{2}$

Cash flow news (column $k = 2$)

Discount rate news (terms a and b, column $k = 1$)

The Bansal-Yaron figure

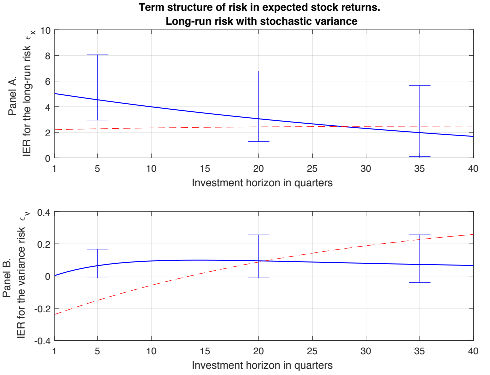

Figure 2
Term structure of risk in expected stock returns. Bansal and Yaron (2004).
The red dashed lines correspond to the theoretical term structures of risk. The blue solid

- Shock to growth rate of consumption
  - Inferred from shock to P/D; model says P/D is cash-flow news; data disagree
- Shock to conditional variances
  - Data say small positive effect on multiperiod returns — next slide

$\mathcal{IER}$ as a diagnostic tool?

- Inconsistent "advice" across examined models
  - Bansal/Yaron: empirically, positive shocks to conditional vols correspond to positive return shocks at all horizons
  - Drechsler/Yaron: empirically, positive shocks to conditional vols, time-varying mean of conditional vols, and jump in vols all correspond to *negative* return shocks at all horizons
- Possible partial explanation: estimated models disagree about the main drivers of P/D
  - P/D tends to pick up otherwise unobserved state variables

    Example: P/D falls and conditional volatilities do not change—must be drop in expected consumption growth in Bansal/Yaron and increase in expected volatility growth in Drechsler/Yaron

Final comments

- Term structure of shocks to expected returns is a nice idea that will benefit from
  - Tighter intuition; if not cash flow news versus discount rate news, what is it?
  - Model-free, or less model-dependent, stylized facts about $\mathcal{IER}$s for different types of shocks