
Term structure of risk in expected returns

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- Macro-based asset pricing literature in a nutshell
 - Many stylized facts about asset prices and returns
 - Standard menu of aggregate shocks

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- Rashomon effect?...

The Rashomon effect is not only about differences in perspective. It occurs particularly where such differences arise in combination with the absence of evidence to elevate or disqualify any version of the truth, plus the social pressure for closure on question. Anderson (2016)

- Macro-based asset pricing literature in a nutshell
 - Many stylized facts about asset prices and returns
 - Standard menu of aggregate shocks
- Open questions
 - Empirical properties of aggregate shocks
 - Size, frequency, persistence
 - Propagation in asset returns (also cash flows, SDF)
 - Are equilibrium models consistent with these properties?
- This paper offers an empirical methodology to answer these questions
 - Defines and describes the term structure of risk in expected buy-and-hold returns
 - Explores return predictability



- New empirical facts for general equilibrium models
 - Multi-period risk-return tradeoff
- Policy analysis
 - Climate change research (e.g., Bansal, Kiku, and Ochoa, 2015)
 - Monetary policy and asset prices (e.g., Gallmeyer, Hollifield, Palomino, and Zin, 2007)
 - Fiscal policy and asset prices (e.g., Liu, 2016)
 - Innovation and asset prices (Croce, Nguyen, Raymond, and Schmid, 2017)



1. Identification: What economic shocks drive asset returns?

- Classical predictive regressions
 - Observable predictors, e.g., from accounting identities (Campbell-Shiller decomposition)
- Equilibrium theory
 - Predictive variable is a function of state variables
 - Candidate states: consumption surplus ratio, expected consumption growth, economic uncertainty, default probability, consumption disaster probability, etc...

2. Incremental expected return IER

- Suitable for analyzing normal and nonnormal shocks
- Predecessor in asset pricing is a *Shock Elasticity* (Borovicka and Hansen, JoE 2014)
- Predecessor in macroeconomic literature is an *Impulse Response* (Sims, ECMA 1980)



- Two complementary approaches
 1. Cross-sectional approach
 - Information in the cross-section of claims
 - Term structure of risk premia and term structure of sharpe ratios (Bansal, Miller, Yaron (2017), Binsbergen, Brandt, Koijen (2012), Binsbergen, Hueskes, Koijen, Vrugt (2014), Dew-Becker, Giglio, Le, Rodriguez (2016), etc, etc)
 2. Time-series approach
 - Predictability of asset returns
 - This paper
 - A granular approach suitable for policy analysis

1. Identification of economic sources of time-variation in asset returns
2. Incremental expected return
3. Evidence: Term structure of risk in expected stock returns



- Empirical model of the stock return dynamics
Collect stock returns and state variables in one vector
 $Y_t = (\log r_{t-1,t}, s_t)'$ and posit

$$Y_{t+1} = F + Gs_t + W_{t+1}$$

What shocks drive stock returns?

- Empirical model of the stock return dynamics

Collect stock returns and state variables in one vector

$Y_t = (\log r_{t-1,t}, s_t)'$ and posit

$$\begin{aligned} Y_{t+1} &= F + Gs_t + W_{t+1} \\ \log pd_t &= A + Bs_t \\ W_{t+1} &= \Gamma \underbrace{z_{t+1}}_{\text{Jumps}} + (C + Ds_t)^{1/2} \underbrace{\varepsilon_{t+1}}_{\text{Gaussian}}, \end{aligned}$$

where $\Pr(z_{t+1}) = \Lambda_0 + \Lambda s_t$.





$$IER(r_{t,t+\tau}, \text{shock}_{t+1}, I_t) = \log E(r_{t,t+\tau} | I_t, \text{shock}_{t+1}) - \log E(r_{t,t+\tau} | I_t)$$

- Implementation

1. Add extra amount of risk to condition on a shock
2. Use the law of iterated expectations to take into account how a shock propagates in the future

Example

- A model with $s_t = \lambda_t$

$$\log r_{t,t+1} = r + \gamma_r Z_{t+1}$$

$$\lambda_{t+1} = (1 - \nu_\lambda) \nu_\lambda + \nu_\lambda \lambda_t + \sigma_\lambda \varepsilon_{\lambda_{t+1}} + Z_{t+1}$$

- Two sources of return variation
 - $\varepsilon_{\lambda_{t+1}} \sim \mathcal{N}(0, 1)$
 - $z_{t+1} | \rho_{t+1} \sim \text{Gamma}(\rho_{t+1}, \theta)$, $\rho_{t+1} \sim \text{Poisson}(h_\lambda \lambda_t)$

Example cont'd

$$IE\mathcal{R}(r_{t,t+\tau}, \text{shock}_{t+1}, I_t) = \log E(r_{t,t+\tau} | I_t, \text{shock}_{t+1}) - \log E(r_{t,t+\tau} | I_t)$$

- As in any affine model,

$$\log E_t r_{t,t+\tau} = \mathcal{A}_0(\tau) + \mathcal{A}_\lambda(\tau)\lambda_t$$

- Use the law of iterated expectations to represent

$$\begin{aligned} \log E_t r_{t,t+\tau} &= \log E_t(r_{t,t+1} \cdot (E_{t+1} r_{t+1,t+\tau})) \\ &= \log E_t(a + b\lambda_t + \mathcal{A}_\lambda(\tau-1)\sigma_\lambda \varepsilon_{\lambda_{t+1}} + (\gamma_r + \mathcal{A}_\lambda(\tau-1))z_{t+1}) \end{aligned}$$

where

$$\begin{aligned} a &= r + \mathcal{A}_0(\tau-1) + \mathcal{A}_\lambda(\tau-1)(1 - v_\lambda)v_\lambda, \\ b &= \mathcal{A}_\lambda(\tau-1)v_\lambda \end{aligned}$$

- Condition on the shock at $t+1$ to compute the $IE\mathcal{R}$
 - $\tilde{\varepsilon}_{\lambda_{t+1}} = \varepsilon_{\lambda_{t+1}} + \Delta_\lambda$ and $IE\mathcal{R}(r_{t,t+\tau}, \varepsilon_{\lambda_{t+1}}, \lambda_t) = \mathcal{A}_\lambda(\tau-1)\sigma_\lambda \Delta_\lambda$

Example cont'd

$$I\mathcal{E}\mathcal{R}(r_{t,t+\tau}, \text{shock}_{t+1}, I_t) = \log E(r_{t,t+\tau} | I_t, \text{shock}_{t+1}) - \log E(r_{t,t+\tau} | I_t)$$

- Recall

$$\log E_t r_{t,t+\tau} = \log E_t (a + b\lambda_t + \mathcal{A}_\lambda(\tau - 1)\sigma_\lambda \varepsilon_{\lambda t+1} + (\gamma_r + \mathcal{A}_\lambda(\tau - 1))z_{t+1})$$

- Two sources of randomness in z_{t+1}
 - Occurrence of the shock $p_{t+1} \sim \text{Poisson}(h_\lambda \lambda_t)$
 - Size of the shock $z_{t+1} | p_{t+1} \sim \text{Gamma}(p_{t+1}, \theta)$

Example cont'd

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- I use an insight from Gourieroux and Jasiak (2006) and represent z_t as a process

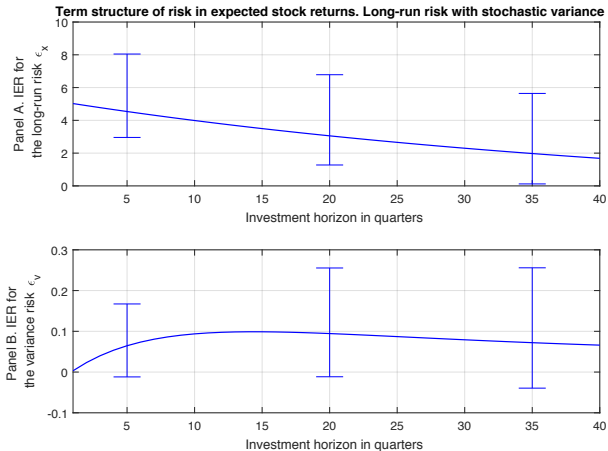
$$z_{t+1} = \theta h_\lambda \lambda_t + (2h_\lambda \lambda_t \theta^2)^{1/2} \varepsilon_{z t+1}$$

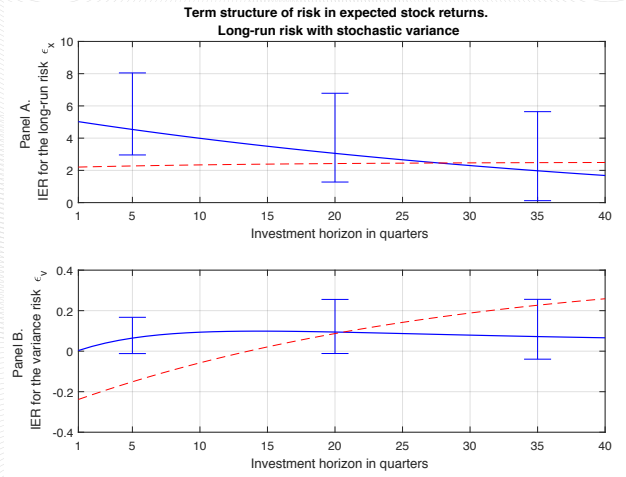
- Characterize shock $\varepsilon_{z t+1}$
 - $\tilde{\varepsilon}_{z t+1} = \varepsilon_{z t+1} + \Delta_z$ and

$$I\mathcal{E}\mathcal{R}(r_{t,t+\tau}, \varepsilon_{z t+1}, \lambda_t) = (2\lambda_t \theta^2)^{1/2} (\gamma_r + \mathcal{A}_\lambda(\tau - 1)) \Delta_z$$



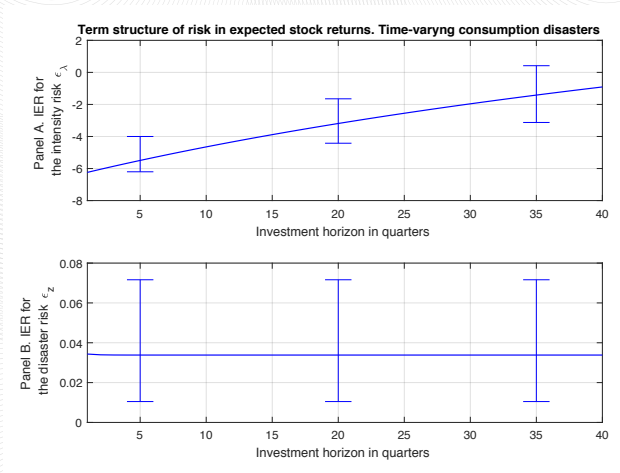
- Data: consumption growth, returns, price-dividend ratio from NIPA BEA and CRSP over 1947:II to 2015:IV
- Empirical model
 - Joint dynamics of consumption growth, returns, and latent states
 - Additional observation equation: price-dividend ratio is an affine function of the state vector
- Bayesian MCMC
 - Estimate latent states
 - Identify structural shocks: observation equation + zero restriction – dividend shock does not contemporaneously affect consumption

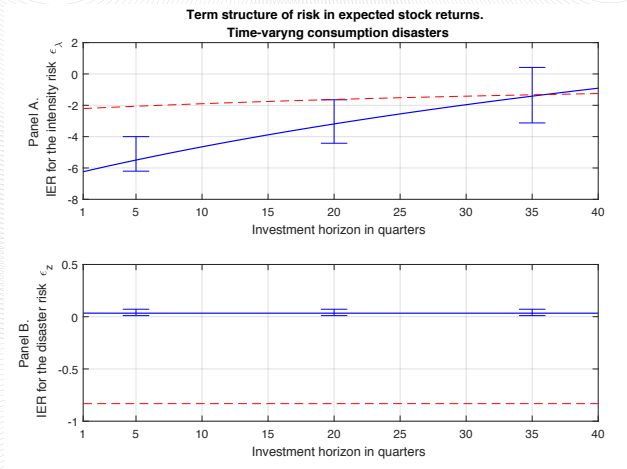






- To marry the model with the data
 - IES is negative
 - A positive long-run risk shock increases marginal utility
 - The leverage parameter is less than one
- The source of discrepancy
 - A downward sloping term structure of real yields

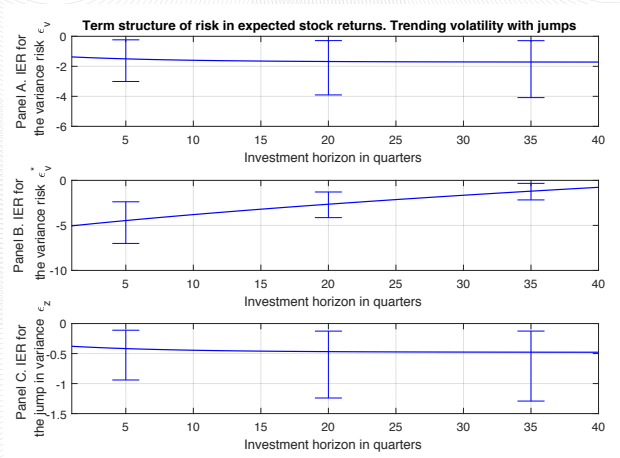




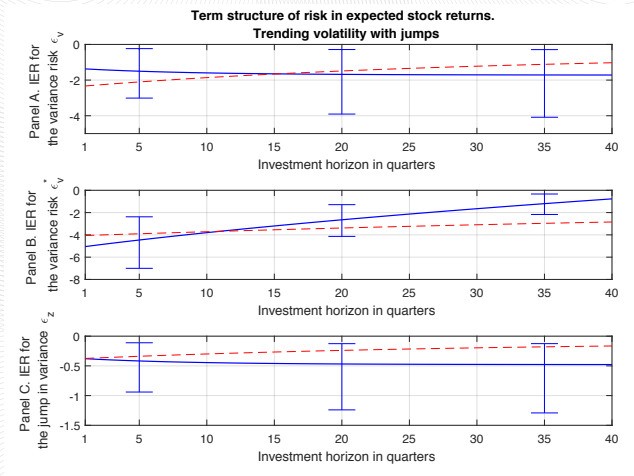
► Estimated dynamics



- To marry the model with the data
 - The leverage parameter is less than zero
- The source of discrepancy
 - Big negative movements in returns and consumption are not contemporaneous



► Estimated dynamics





- Variance shocks can generate realistic magnitudes of risk premium and time variation in expected returns
- Long-run variance shock is a key driver of return predictability



- Predictability is a powerful source of information about the term structure of risk in asset returns
- Characterization of the term structure of nonnormal sources of risk
- Volatility risk may play a more prominent role than we thought before

- Formal definition and mathematical formulation in Borovicka, Hansen, Hendricks, Scheinkman (2011) and Borovicka and Hansen (2014)

$$\ell(r_{t,t+\tau}, \varepsilon_{vt+1}, v_t) = \frac{E_t(r_{t,t+\tau} \cdot \varepsilon_{vt+1})}{E_t r_{t,t+\tau}} = \tilde{E}_t \varepsilon_{vt+1}.$$

where $E_{t+1}(r_{t,t+\tau})/E_t(r_{t,t+\tau})$ determines the change of measure

- In this example

$$\begin{aligned}\ell(r_{t,t+\tau}, v_t, \varepsilon_{rt+1}) &= \gamma_r(1 + \mathcal{A}_r(\tau - 1))v_t^{1/2}, \\ \ell(r_{t,t+\tau}, v_t, \varepsilon_{vt+1}) &= \mathcal{A}_v(\tau - 1)\sigma_v.\end{aligned}$$

