Equilibrium Activism*

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Abstract

We study a general equilibrium production economy with a financial sector that contains both actively-managed and passively-managed funds. Activists in the actively-managed sector are socially valuable because they improve productive efficiency by monitoring firm managers; i.e., through the use of “voice.” Quants in the actively-managed sector displace activists by buying previously identified efficient firms. A representative household allocates its wealth to activist, quant, and index funds. We examine a steady state equilibrium where actively and passively managed sectors coexist in positive net supply. Given the model structure, we conduct a series of experiments to characterize the impact on the steady state equilibrium of changes in the matching function between active managers and firms and changes in the costs of intermediation.

Keywords: Activism, delegated fund management, index funds

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1 Introduction

If a firm’s managers are destroying shareholder value, an asset manager faces two primary choices: voice or exit. Voice, or activism, is generally understood to describe a broad range of shareholder actions designed to influence a firm’s real or financial policy decisions. As such, it is recognized as one important component of corporate governance. We build a model of the asset management industry, embedded in general equilibrium, with “activism” at its heart. Our goal is to understand the underlying economic forces that drive the aggregate level of activism in the economy.

The model economy starts with a set of productive technologies or “firms.” The firms are owned by a household sector that is characterized by a representative agent. The household invests in firms through three different types of intermediaries: “activists,” “quants,” and a passive index.

An activist creates a “fund” by matching with a firm, exercising “voice,” and forcing the firm to use its capital efficiently. The firm remains efficient as long as it is matched with an activist. A quant manager can identify and invest in an efficient firm, but the quant does not engage in using voice. Although the quant fund does not offer the household access to the efficiently run firm at a lower cost than the activist, the arrival of the quant destroys the match between the activist. This is not considered explicitly in the model, but we think of this exogenous feature of the model as a proxy to some unobserved private benefit that the activist loses when the quant matches to the firm. The quant fund is effectively serving the role of the free-rider in Shleifer and Vishny [1986]. In the absence of a match with an activist, a firm will revert over time, at some rate, to an inefficient use of its capital.

The passive fund allows the representative household access, at a low cost, to an equal-weighted portfolio of all firms. The existence of the passive fund captures the intuition that the household may not require skilled managers to spread capital evenly across firms, but they
do required skilled managers to identify the most efficient funds. The household determines the demand for each of the three types of funds by solving an optimal consumption and portfolio choice problem. In equilibrium, the markets for goods and fund shares all clear, determining the aggregate size of the sector of each fund type in the economy.

For a wide range of parameter values, there is a non-degenerate stationary distribution for each manager type in the steady state of the model. The endogenous fees for the activists and quants are identical because, from the household’s perspective, the two non-passive manager types provide the same economic good: higher expected returns (at the cost of reduced portfolio diversification). Although the equilibrium characteristics of activists and quants are symmetric in many regards, their responses to some changes to equilibrium parameters are not necessarily identical. There is a “predator/prey” component to the dynamics of the relative sizes of the activist and quant sectors that can lead to some interesting interaction effects.

Given the solution to the model, we are able to perform a number of experiments that examine the key factors determining the interactions of activists and quants and of households with both of these types of active managers. In particular, how do search costs and fee structures affect the relative sizes of the active management sectors and market efficiency? How sensitive are the sizes of active and passive markets to changes in household demand for diversification benefits offered by active managers? Finally, what can we learn about the relationship between active management and household welfare?

In addition to demonstrating that a non-degenerate distribution of activists and quants co-exists in equilibrium, our baseline parameters demonstrate how fund market competition impacts assets under management for the household through diversification and spillover effects. As the number of activist funds grow, the household invest more in the sector overall as it becomes better diversified. However, more managed funds also make the passive index more efficient leading to a spillover effect that eventually makes assets under management...
in the active sector fall. Quants also generate a spillover effect. If more of them are in the
market, activist assets under management fall even sooner. This dynamic between activists
and quants also impacts differentially their present value of fees collected. From a welfare
perspective, households actually prefer an environment where activists and quants jointly
make the market more efficient as they cannot stop the free entry of quants.

The dynamics of the model are particularly sensitive to variations in the cost of interme-
diation. If quant search costs fall too much, say through better technology to detect efficient
firms, the size of the activist sector can crash ultimately leading to a crash in the quant
sector too. This renders the market less efficient leading to lower household welfare. Across
the board increases in the cost of intermediation through a fee on the passive index can also
adversely impact activists relative to quants again leading to lower household welfare.

In focusing on the role of voice in equilibrium, we have largely abstracted from the
conventional focus of mutual fund and hedge fund research: does active management deliver
alpha through the selection of mispriced securities? We say “largely” because quant managers
do engage in a form of identification of mispriced assets. Our purpose is not to argue that
the conventional focus is unimportant but rather to argue that activism may play a more
important role in understanding the role of asset managers in general equilibrium than has
generally been acknowledged. This view is consistent with the recent survey evidence in
Mc Cahery et al. [2016].

The rest of the paper is organized as follows: After placing our work in the context of
the relevant (large) literature on both shareholder monitoring and active management, we
present the basic model structure and then describe the equilibrium of the model. We then
examine how the model equilibrium responds to practically relevant shocks, and we conclude
in the final section of the paper.
2 Our Contribution Relative to the Existing Literature

In order for any shareholder (or group of shareholders) to effectively monitor firm managers, they must solve a free-rider problem; i.e., the costly efforts of shareholders who monitor the firm also benefit other shareholders who exert no effort. This problem might be overcome by large shareholders via takeovers, as in Shleifer and Vishny [1986]. Yet ownership that is too highly concentrated could lead to sub-optimally tight control by shareholders [Burkart et al., 1997], or to weaker monitoring incentives stemming from reduced liquidity and price informativeness [Holmström and Tirole, 1993]. Assuming that the right number of shareholders are paying the right amount of attention and providing improved incentives to the firm’s managers remains a nontrivial problem [Core et al., 2003].

The results in Admati, Pfleiderer, and Zechner [1994] provide theoretical arguments against the use of voice by demonstrating that the equilibrium level of monitoring is well below the socially optimal level of monitoring, and and DeMarzo and Urosevic [2006] extend this result to show that, over time, an activist will actually hold a perfectly diversified portfolio. Marinovic and Varas [2019] extend these findings by showing that whether or not the activist monitors and whether or not that monitoring increases firm value depends critically on the presence of information asymmetry about the activist’s ability.¹

Despite these substantial incentive and information obstacles, McCahery et al. [2016] present survey evidence that over half of institutional investors engage directly with the management and boards of the firms in which they hold stock, presumably to the benefit of shareholders at large. We reconcile the theoretical results with the empirical findings by abstracting from the specific details of how activists change firm policies – and by extension firm value. Instead, we assume a matching technology that pairs fund managers (both ac-

¹This additional layer of delegation introduces new agency problems that have been the focus of a large literature on optimal managerial contracting; see, for example, the theoretical work of Bhattacharya and Pfleiderer [1985], Stoughton [1993], Heinkel and Stoughton [1994], Starks [1987], and the empirical work in Almazan et al. [2004].
tivists and quants) to units of productive capital. Once the match is formed the monitoring requires no costly effort by managers, and the representative household has perfect information about which units of capital are matched to each type of manager. We use voice and monitoring as a metaphor for how active managers can deliver value to households in equilibrium in the absence of asymmetric information. We are interested in understanding the factors that affect the relative sizes of the active and passive sectors of the asset management industry and how the structure of the asset management industry affects household welfare in general equilibrium under perfect information.

Pastor and Stambaugh [2012] provides an alternative explanation for the relative sizes of the active and passive sectors of the fund industry. Their focus is on explaining the poor performance of active funds relative to a passive benchmark (the “active management puzzle”) in an equilibrium model where funds face decreasing returns to scale; i.e., when fund performance erodes because too many managers trade in related strategies. Their explanation of active vs. passive sector sizes is complementary to our model. Their model is more general than ours in the sense that “alpha” in their analysis might come from a variety of manager actions whereas we focus exclusively on voice. Our model is more general than theirs in that we focus on a production economy and our household explicitly solves a multiperiod (infinite-horizon) optimization problem. Finally, in Pastor and Stambaugh [2012], investors are learning about managerial skill while our framework assumes complete information.

We assume that passive managers provide diversification services at low cost, and they do not engage in any use of voice or (by definition) other active strategies. Azar et al. [2018] purport to provide evidence that even passive managers can facilitate collusion in concentrated industries, to the detriment of households. However, there is a growing literature that contradicts the findings in Azar et al. [2018]; see, for example Dennis et al. [2018].
3 Model

We study a continuous-time, infinite-horizon economy in which the fund management industry intermediates between households and productive investment opportunities. We build the model from the bottom up, beginning with the productive technologies, or firms. Investment managers who are either activists or quants search for opportunities to match with firms, thereby forming funds. A lower cost passive index fund also exists which invests equally in all firms. Finally, a representative household optimally allocates capital across funds. Having closed the model, we solve and analyze fund and capital market dynamics in general equilibrium.

3.1 Firms

Capital may be productively invested in $N$ firms. All firms have identical productivity of capital $\mu$, such that a firm $j$ with capital $K_{j,t}$ produces output at gross rate $\mu K_{j,t} dt$. However capital in firm $j$ also depreciates at a rate

$$ K_{j,t} [-\delta_{j,t} dt + \sigma d\bar{W}_t + \sigma dW_{j,t}] . $$

The Brownian motion $\bar{W}_t$ captures a capital depreciation shock common to all firms, whereas the Brownian motion $W_{j,t}$ captures firm-specific depreciation, which is independent of $\bar{W}_t$ or $W_{i,t}$, $i \neq j$. At any given time, the deterministic depreciation rate $\delta_{j,t}$ takes one of two values: $\delta_{j,t} = \underline{\delta}$ if the firm is efficiently run, or $\delta_{j,t} = \bar{\delta} > \underline{\delta}$ otherwise. This simple reduced-form specification allows for activism to have a role in the fund management sector.
3.2 Funds

Activists seek out inefficient firms to monitor, thereby rendering them efficient. Firms are efficient as long as they are combined with an activist. How these managers effect change or “voice” is modeled in reduced form. However, we have in mind mechanisms for voice as discussed in the survey evidence in McCaher et al. [2016]. We also assume that activism is always successful. If an activist chooses to invest in a firm, that firm is always efficiently run as modeled by the lower depreciation rate \( \delta_{j,t} = \delta^2 \).

By contrast quantitative fund managers, or quants, are able to invest in efficient firms that are currently monitored by activists. Quants perform no monitoring themselves: instead they displace incumbent activists. Once the activist stops monitoring the firm, there is a chance that it reverts to inefficiency. Quant managers are the manifestation, in the model, of the free-rider problem of Shleifer and Vishny [1986].

A large but finite number of ex-ante identically skilled managers search for opportunities in the labor market for asset managers. Managers match with firms to form investment funds, which may be either quant funds or activist funds. A fund consists of one firm and one manager. An unemployed (potential) manager may search for opportunities as either an activist or a quant, or may choose not to search at all. The choice is sensitive to the current competitiveness of the fund market: there are \( n_t \) incumbent activist funds, and \( m_t \) incumbent quant funds. Potential managers who choose not to search can be thought of as remaining in the general household pool, earning reservation utility with certainty equivalent value \( cK_t \), where \( K_t \) is the aggregate capital stock, or equivalently aggregate wealth.\(^3\)

If a potential manager chooses to search, he pays flow cost \( \zeta_A K_t dt \) while searching for

\(^2\)As noted earlier, reality dictates that activism will not always be successful and can be curtailed by free rider problems [Grossman and Hart, 1980, Shleifer and Vishny, 1986] or liquidity issues [Coffee, 1991].

\(^3\)The number of potential managers is not generally important to the setup, but we assume that there are a large number of potential managers relative to firms.
activist opportunities, or $\zeta_Q K_t dt$ for quant opportunities. New matches are formed at rates

$$\theta_A \hat{n}_t^{1-\nu} (N - n_t - m_t)\nu,$$  \hspace{1cm} (2)

$$\theta_Q \hat{m}_t^{1-\nu} n_t\nu,$$  \hspace{1cm} (3)

for activists and quants, respectively, where $\hat{n}_t$ is the number of potential managers searching for activist opportunities and $\hat{m}_t$ the number of potential quants searching. In our experiments, below, we assume that $\nu = 1/2$ and that there is no difference in $\nu$ between activists and quants. This corresponds to a prior belief that matching is equally difficult for both manager types.

Constant returns to scale imply that individual potential managers find matches at rates

$$\theta_A \left( \frac{N - n_t - m_t}{\hat{n}_t} \right)^\nu,$$  \hspace{1cm} (4)

$$\theta_Q \left( \frac{n_t}{\hat{m}_t} \right)^\nu.$$  \hspace{1cm} (5)

That is, the chance a searching manager succeeds depends on the tightness of his labor market, a ratio of available target firms to searching managers. Note that the rate at which new quant funds are formed is also the rate at which existing activist funds dissolve. Finally, quant funds dissolve at an exogenous rate $\theta_L$ individually, or $\theta_L m_t$ in the aggregate. When a quant fund dissolves, an efficient firm reverts to being inefficient.

Potential managers take their earnings present value as given when making search decisions. An incumbent activist manager earns fee income with time $t$ present value $\Phi(n_t, m_t, K_t)$. An incumbent quant manager has present value of expected earnings $\Psi(n_t, m_t, K_t)$. Fee income depends on funds under management, which reflects household asset allocation, for which we later solve in equilibrium.

Potential managers will enter the labor market (search) until the net present value of a
new fund is zero. For activists, \( \hat{n}_t \) is the maximum number of searchers such that

\[
(\Phi(n_t + 1, m_t, K_t) - \xi K_t) \theta_A \left( \frac{N - n_t - m_t}{n_t} \right) \geq \zeta A K_t. \tag{6}
\]

When a potential activist matches with a target firm, he exchanges fee revenues for his household consumption stream. The exchange must be favorable enough, and the intensity of matching high enough, to justify the search costs. The equivalent condition for quants is

\[
(\Psi(n_t + 1, m_t, K_t) - \xi K_t) \theta_Q \left( \frac{n_t}{m_t} \right) \geq \zeta Q K_t. \tag{7}
\]

We later show that the present value of fees is homogeneous of degree one in capital \( K_t \), such that they can be written \( \Phi(n_t, m_t, K_t) = \phi(n_t, m_t) K_t \) for activists, and \( \Psi(n_t, m_t, K_t) = \psi(n_t, m_t) K_t \) for quants. This allows us to solve for \( \hat{n}_t \), the number of potential managers seeking to start activist funds, as the largest integer such that

\[
\hat{n}_t \leq \left( (\phi(n_t + 1, m_t) - \xi) \frac{\theta_A}{\zeta A} \right)^{\frac{1}{\nu}} (N - n_t - m_t). \tag{8}
\]

whereas \( \hat{m}_t \), the number of potential managers searching to be quants, is the largest integer such that

\[
\hat{m}_t \leq \left( (\psi(n_t, m_t + 1) - \xi) \frac{\theta_Q}{\zeta Q} \right)^{\frac{1}{\nu}} n_t. \tag{9}
\]

Finally, a third type of investment fund exists alongside quants and activists: a passive index fund, which invests equally in each of the \( N \) firms. This fund is unique, requires no manager, and charges a small, exogenous, and constant fee rate \( \bar{\pi} \) proportional to capital under management.\(^4\) In addition to capturing competition from low-cost index funds, the passive fund reflects the idea that households do not require skilled managers in order to spread capital evenly across all firms, but they do require skilled managers to identify the

\(^4\)In our baseline parameters, it is zero.
most efficient firms. In a simple way, this captures the idea that fund managers are better informed than households.

3.3 Households

A representative household allocates capital to investment funds in order to maximize expected lifetime utility. While the state of competition among funds matters to households, there is no reason for households to differentiate between funds of a given type. For example, each activist charges an identical fee proportional to capital, and each invests in a unique — but equivalent — efficient firm. Hence it is optimal, for purposes of diversification, for the household to invest the same amount in each activist fund. The relevant question is how much the household should invest in activist funds collectively.

Without loss of generality, let the first \( n_t \) firms be activist funds, the next \( m_t \) firms be quant funds, and the remaining \( N - n_t - m_t \) firms belong exclusively to the the passive index.\(^5\) We define composite Brownian motions

\[
W_{A,t} = \sum_{j=1}^{n_t} W_{j,t}, \quad (10)
\]

\[
W_{Q,t} = \sum_{j=n_t+1}^{n_t+m_t} W_{j,t}, \quad (11)
\]

\[
W_{I,t} = \sum_{j=n_t+m_t+1}^{N} W_{j,t}, \quad (12)
\]

representing risk specific to activists, quants, and the passive fund, respectively.

While we have in mind that the passive fund fee \( \bar{\pi} \) is small, a fraction \( \frac{N-n_t-m_t}{N} \) of its holdings are inefficient, with higher depreciation rate \( \bar{\delta} \). Both activists and quants invest exclusively in efficient firms, with deterministic capital depreciation \( \bar{\delta} \). However these funds

\(^5\)Recall that funds consist of non-overlapping firm-manager pairs, but the passive index invests equally in every firm.
also charge commensurately higher fees proportional to capital under management. Fee rates may vary depending on the amount of competition in the fund market. The quant’s fee rate is \( \bar{\psi}_t > 0 \), and the activist’s fee rate is \( \bar{\phi}_t \). Households and potential managers take these fees as given. Later we explain how incumbent managers strategically set fees to maximize revenues.

Households allocate aggregate capital, \( K_t \), across the three types of funds, and also choose the consumption-capital ratio \( c_t \). Let \( a_t \) be the activists’ fraction of capital, \( q_t \) the quants’ fraction of capital, and the residual \( 1 - a_t - q_t \) is invested in the passive index. The aggregate capital accumulation process is

\[
\frac{dK_t}{K_t} = (\mu - c_t)dt + \sigma dW_t
\]

\[
+ \begin{bmatrix} a_t \\ q_t \\ (1 - a_t - q_t) \end{bmatrix} \cdot \begin{bmatrix} - (\bar{\phi}_t + \delta) dt + \frac{\sigma}{\sqrt{m}} dW_{A,t} \\ - (\bar{\psi}_t + \delta) dt + \frac{\sigma}{\sqrt{m}} dW_{Q,t} \\ -(\bar{\pi} + \delta_{I,t}) dt + \frac{\sigma}{N} \left( \sqrt{N - n_t - m_t} dW_{I,t} + \sqrt{n_t} dW_{A,t} + \sqrt{m_t} dW_{Q,t} \right) \end{bmatrix},
\]

\[(13)\]

where

\[
\delta_{I,t} = \frac{1}{N} \left( (N - n_t - m_t) \delta + (n_t + m_t) \bar{\delta} \right)
\]

\[(14)\]

is the mean depreciation rate of firms in the index.

Households have CRRA utility, and choose consumption and capital allocation to maximize their lifetime expected utility,

\[
\max_{\{c_t, a_t, q_t\}_{t=0}^\infty} E_0 \left[ \int_0^\infty e^{-\rho t} u(c_t K_t) dt \right],
\]

\[(15)\]

subject to Equation (13).
4 Equilibrium

The economic state is summarized by the tuple \( \omega_t = (m_t, n_t) \), a Markov process capturing current competition in the fund markets, and by the aggregate capital stock \( K_t \). Since the economy is time-homogeneous, we omit time subscripts in the solution.

4.1 The Fund Managers’ Problems

We begin by solving for a manager’s entry decision taking fees as given. Expectation under the risk-neutral measure is denoted \( \mathbb{E}^Q \), and the instantaneous risk-free discount rate is \( r(w) \), i.e., a function of the Markov state only. Define the space of Markov states \( \Omega \), and for current state \( \omega \in \Omega \), the instantaneous rate of transition to \( \omega' \in \Omega \) is \( \lambda_{\omega,\omega'} \), and \( \Lambda \) the transition matrix, with \( \lambda_{\omega,\omega'} = -\sum_{\omega''\in\Omega, \omega''\neq \omega} \lambda_{\omega,\omega''} \). The equilibrium fee and the manager’s entry decision are determined simultaneously, but we discuss them sequentially.

The instantaneous hazard rate \( \beta(\omega) \), serves two distinct purposes in the model: (i) It captures the possibility that a particular activist is driven out of the market. This occurs when the activist’s firm becomes part of a quant fund. (ii) The hazard rate also limits the conditions under which potential activists will search to form new funds. It is possible for the present value of activist fees to be positive even if the current flow of fees is non-positive, i.e., if the household would choose a zero or short position in activist funds. To prevent this, we impose that activists immediately exit if their current fee flow turns negative. It follows that

\[
\beta(\omega) = \begin{cases} \frac{\lambda_{(m,n),(m+1,n-1)}}{n} & \text{if } a(\omega) > 0, \\ \infty & \text{otherwise}. \end{cases} \tag{16}
\]

The present value of an incumbent activist’s fees is

\[
\Phi(\omega_t, K_t) = \mathbb{E}_t^Q \left[ \int_t^\infty e^{\int_t^s \beta(\omega_v) + r(\omega_v)dv} \frac{\phi(\omega_s) a(\omega_s)}{n_s} K_s ds \right]. \tag{17}
\]
We use a generalized Feynman-Kac theorem to express the value of the stochastic integral above as the solution to a PDE, which reduces to a simple system of equations in the Markov state variable due to homogeneity of the valuation in $K$.\footnote{See Zhu et al. [2015], Theorem 3.2.} Let $A$ be the infinitesimal generator for $(\omega_t, K_t)$. Then the function $\Phi$ satisfies

$$A\Phi(\omega, K) - (\beta(\omega) + r(\omega))\Phi(\omega, K) + \frac{\bar{\nu}(\omega) a(\omega)}{n} K = 0. \quad (18)$$

Recall our supposition $\Phi(\omega, K) = \phi(\omega)K$. Note that $\Phi_{KK} = 0$ and the drift of $dK_t$ is $(r(\omega_t) - c(\omega_t))K_t$ under the risk-neutral measure, where $c(\omega_t)$ is the aggregate consumption-capital ratio, interpretable as a dividend yield. Following equation (2.9) in Zhu et al. [2015], we have

$$A\Phi(\omega, K) = \phi(\omega)(r(\omega) - c(\omega))K + \sum_{\omega' \in \Omega} \lambda_{\omega, \omega'} \left[ \phi(\omega') - \phi(\omega) \right] K. \quad (19)$$

The optimal $\phi(\omega)$ solves

$$\phi(\omega)(r(\omega) - c(\omega))K + \sum_{\omega' \in \Omega} \lambda_{\omega, \omega'} \left[ \phi(\omega') - \phi(\omega) \right] K $$

$$- (\beta(\omega) + r(\omega))\phi(\omega)K + \frac{\bar{\nu}(\omega) a(\omega)}{n} K = 0,$$

$$\Rightarrow \sum_{\omega' \in \Omega} \lambda_{\omega, \omega'} \left[ \phi(\omega') - \phi(\omega) \right] - (\beta(\omega) + c(\omega))\phi(\omega) + \frac{\bar{\nu}(\omega) a(\omega)}{n} = 0. \quad (21)$$

The solution for $\Psi(\omega, K) = \psi(\omega)K$, the present value of quant fees, is similar. Recall that a quant exits the market if his firm reverts to inefficiency, hence his hazard rate is $\ell L$.

An incumbent quant has fee present value

$$\Psi(\omega_t, K_t) = E^Q_t \left[ \int_t^\infty e^{\ell L s + r(\omega_s)} ds \psi(\omega_s) q(\omega_s) m_s K_s ds \right]. \quad (22)$$
Therefore the function $\Psi$ satisfies

$$A\Psi(\omega, K) - (\theta_L + r(\omega))\Psi(\omega, K) + \frac{\bar{\psi}(\omega)q(\omega)}{m}K = 0,$$

(23)

where

$$A\Psi(\omega, K) = \psi(\omega)(r(\omega) - c(\omega))K + \sum_{\omega' \in \Omega} \lambda_{\omega,\omega'}[\psi(\omega') - \psi(\omega)]K.$$

(24)

After simplification this leaves

$$\sum_{\omega' \in \Omega} \lambda_{\omega,\omega'}[\psi(\omega') - \psi(\omega)] - (\theta_L + c(\omega))\psi(\omega) + \frac{\bar{\psi}(\omega)q(\omega)}{m} = 0.$$

(25)

Subject to $\phi(\omega)$ and $\psi(\omega)$ satisfying Equation (21) and Equation (25), respectively, managerial search intensities $\hat{n}$ and $\hat{m}$ satisfy Equation (8) and Equation (9), respectively.

Therefore instantaneous transition rates are

$$\lambda_{(m,n),(m,n+1)} = \theta_A\hat{n}^{1-\nu}(N - n - m)^\nu,$$

(26)

$$\lambda_{(m,n),(m+1,n-1)} = \theta_Q\hat{m}^{1-\nu}n^\nu,$$

(27)

$$\lambda_{(m,n),(m-1,n)} = \theta_Lm,$$

(28)

and zero to all other states, subject to the additional restriction that transition intensity to states outside of $\{0 \ldots N\}$ is always zero.

Given the entry decision, we now endogenize how fee rates are set by incumbent managers. We assume managers behave strategically, albeit in a limited sense. For each state $\omega$, we solve for a symmetric Nash equilibrium in which each manager chooses his fee rate to maximize his flow of revenues, i.e., the product of his fee rate and the household’s allocation to his fund. In doing so the manager takes into account the response of the household to a potential change in fee rate, given the fees charged by the manager’s competitors. However incumbent managers
do not consider how their fee strategy alters fund market dynamics through its effects on the labor market search behavior of potential managers. This assumption is primarily for tractability, since it decouples the determination of fee rates from equilibrium fund market dynamics. However it seems reasonable that while fund managers might respond to current competitive pressures, they may not anticipate how their behavior will alter the evolution of the fund market in the future.

For some state \( \omega = (m, n) \) with \( n > 0 \), consider an individual activist who chooses his fee rate \( \bar{\phi}'(\omega) \), while the remaining \( n - 1 \) activists charge fee \( \bar{\phi}(\omega) \) and \( m \) quants charge fee \( \bar{\psi}(\omega) \). Given his choice of fee, the individual activist will attract capital

\[
a'(\omega) = \frac{N(\delta_I(\omega) + \bar{\pi} - \bar{\delta}) - (N - m - n + 1)\bar{\phi}'(\omega) - m\bar{\psi}(\omega) - (n - 1)\bar{\phi}(\omega)}{\gamma \sigma^2 (N - m - n)} \tag{29}
\]

The individual activist solves revenue maximization problem

\[
\max_{\bar{\phi}'(\omega)} \bar{\phi}'(\omega)a'(\omega), \tag{30}
\]

which has solution

\[
\bar{\phi}'(\omega) = \frac{N(\delta_I(\omega) + \bar{\pi} - \bar{\delta}) - m\bar{\psi}(\omega) - (n - 1)\bar{\phi}(\omega)}{2(N - m - n + 1)} \tag{31}
\]

Similarly, for some state \( \omega = (m, n) \) with \( m > 0 \), an individual quant choosing fee rate \( \bar{\psi}'(\omega) \), while \( n \) activists charge fee \( \bar{\phi}(\omega) \) and the remaining \( m - 1 \) quants charge fee \( \bar{\psi}(\omega) \), attracts capital

\[
b'(\omega) = \frac{N(\delta_I(\omega) + \bar{\pi} - \bar{\delta}) - (N - m - n + 1)\bar{\psi}'(\omega) - (m - 1)\bar{\psi}(\omega) - n\bar{\phi}(\omega)}{\gamma \sigma^2 (N - m - n)} \tag{32}
\]
The individual quant solves revenue maximization problem

$$\max_{\psi'(\omega)} \psi'(\omega)b'(\omega),$$

which has solution

$$\bar{\psi}'(\omega) = \frac{N(\delta f(\omega) + \bar{\pi} - \bar{\delta}) - (m - 1)\bar{\psi}(\omega) - n\bar{\phi}(\omega)}{2(N - m - n + 1)}. \tag{34}$$

The symmetric Nash equilibrium is one in which all activists choose identical state-contingent fee $\bar{\phi}(\omega)$, all quants choose identical state-contingent fee $\bar{\psi}(\omega)$, and no individual fund manager has an incentive to diverge from the rate schedule. Therefore, for each state $\omega$, fee rates satisfy

$$\bar{\phi}(\omega) = \frac{N(\delta f(\omega) + \bar{\pi} - \bar{\delta}) - m\bar{\psi}(\omega) - (n - 1)\bar{\phi}(\omega)}{2(N - m - n + 1)}, \tag{35}$$

$$\bar{\psi}(\omega) = \frac{N(\delta f(\omega) + \bar{\pi} - \bar{\delta}) - (m - 1)\bar{\psi}(\omega) - n\bar{\phi}(\omega)}{2(N - m - n + 1)} \tag{36}.$$ 

It is not surprising, given that activists and quants operate funds with identical return distributions, that they will optimally charge identical rates. Equilibrium fee rates are

$$\bar{\phi}(\omega) = \bar{\psi}(\omega) = \frac{N(\delta f(\omega) + \bar{\pi} - \bar{\delta})}{2N - m - n + 1}. \tag{37}$$

### 4.2 The Household’s Problem

The household’s problem, in Equation (15), is essentially a conventional planner’s problem with affine production technologies and no capital adjustment costs. The novelty is that the risk and productivity characteristics of the production technologies arise from equilibrium
in the fund manager labor market. We make the usual conjecture that the value function can be written

\[ V(\omega, K) = \frac{1}{1 - \gamma} H(\omega) K^{1-\gamma}, \]  

for a function \( H(\omega) \) to be determined.

The value function satisfies Hamilton-Jacobi-Bellman (HJB) equation

\[
\frac{1}{1 - \gamma} [\rho H(\omega) - \sum_{\omega' \in \Omega} \lambda_{\omega, \omega'} H(\omega')] \\
= \max_{c, a, q} \left\{ u(c) + (\mu - c - a(\bar{\phi}(\omega) + \bar{\delta}) - q(\bar{\psi}(\omega) + \bar{\delta}) - (1 - a - q)(\bar{\pi} + \delta_I(\omega)) H(\omega) \right. \\ \\
- \gamma H(\omega) \left. \frac{2}{\sigma^2 + \frac{\sigma^2}{N} \left( 1 - 2aq + \left( \frac{N}{n} - 1 \right)a^2 + \left( \frac{N}{m} - 1 \right)q^2 \right) } \right\}.  
\]

The solution to the asset allocation problem is similar to Merton [1969]: it reflects the mean returns and covariance matrix for the funds. Because transitions in the number of fund managers are unpredictable, they do not introduce a hedging component to the asset allocation problem. Optimal \( c, a, \) and \( q \) are

\[
c(\omega) = H(\omega)^{-1/\gamma},  
\]

\[
a(\omega) = \frac{n \left( (N - m)(\delta_I(\omega) + \bar{\pi} - \bar{\phi}(\omega) - \bar{\delta}) + m(\delta_I(\omega) + \bar{\pi} - \bar{\psi}(\omega) - \bar{\delta}) \right)}{\gamma \sigma^2(N - m - n)},  
\]

\[
q(\omega) = \frac{m \left( (N - n)(\delta_I(\omega) + \bar{\pi} - \bar{\psi}(\omega) - \bar{\delta}) + n(\delta_I(\omega) + \bar{\pi} - \bar{\bar{\phi}}(\omega) - \bar{\delta}) \right)}{\gamma \sigma^2(N - m - n)}.  
\]

To condense notation, write the drift and variance of the household’s optimal portfolio

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\(^7\)Our solution of the household’s problem follows Sotomayor and Cadenillas [2009], who establish optimality conditions in a setting similar to Merton [1969], but with market opportunities and utility contingent upon a continuous-time Markov process. See in particular Theorem 3.2, which covers the case where utility is not directly contingent on the Markov state. Although we generalize to a production setting and endogenous investment opportunities, the household’s problem remains very similar to Sotomayor and Cadenillas [2009].
returns, respectively, as

\[
\hat{\mu}(\omega) = \mu - a(\omega)(\bar{c}(\omega) + \bar{d}) - q(\omega)\bar{d}(\omega) + \bar{d} - (1 - a(\omega) - q(\omega))(\delta(\omega) + \bar{d}), \quad (43)
\]

\[
\hat{\sigma}(\omega) = \bar{\sigma} + \frac{\sigma^2}{N} \left( 1 - 2a(\omega)q(\omega) + \frac{N}{n} - 1 \right) a(\omega)^2 + \left( \frac{N}{m} - 1 \right) q(\omega)^2. \quad (44)
\]

The drift function for the household’s optimal portfolio has a straightforward interpretation: In the absence of activists and quants, the aggregate capital stock grows at \( \mu \) less the high depreciation rate of inefficient firms, since all firms are inefficient in the absence of monitoring. The drift term is then adjusted by the allocation to each manager type multiplied by the net return effect of each manager type. The manager’s effect on expected returns consists of the fee paid to the manager in order to achieve the lower depreciation rate. The diffusion similarly shows the effect of allocating household wealth outside of the passive fund. If there were no activists or quants, the diffusion would consist of the diffusion of the aggregate shock, \( \bar{\sigma} \), plus the (common) individual diffusion divided by \( N \).

Substituting the optimal portfolio choice and consumption and making use of the condensed notation, the HJB equation is

\[
\gamma H(\omega)^{(\gamma - 1)/\gamma} + \left( 1 - \gamma \right) \left( \frac{\hat{\mu}(\omega)^2}{2} - \rho \right) H(\omega) + \sum_{\omega' \in \Omega} \lambda_{\omega, \omega'} H(\omega') = 0. \quad (45)
\]

The risk-free rate under the household’s pricing kernel is

\[
r_f(\omega) = \rho + \gamma(\hat{\mu}(\omega) - c(\omega)) - \frac{1}{2} \gamma(1 + \gamma) \hat{\sigma}^2(\omega). \quad (46)
\]

5 Results

We characterize model behavior via numerical examples and comparative statics. Baseline model parameters are given in Table 1. We also consider three different comparative statics:
(1) lower quant search costs relative to activists, (2) a higher cost to invest in the passive fund, and (3) a lower household risk aversion. The first comparative static is meant to capture a scenario where entry by quants is easier relative to activists. The second comparative static is meant to demonstrate the trade off between investing in the passive sector versus the managed money sector. Finally, the last comparative static is meant to capture the role of lower diversification costs on equilibrium quantities.

Our baseline parameters are chosen to be reasonable relative to observed economic quantities. It is worth noting a few of the parameters. First, the difference between an efficient and an inefficient firm is driven by a depreciation spread of $\bar{\delta} - \underline{\delta} = 4\%$. Given the mean productivity of capital $\mu$ is 11\%, being efficiently monitored is economically meaningful. Second, our number of firms, $N = 25$, is chosen for fast equilibrium computational time. Experiments with larger numbers of firms lead to similar results. Finally, the search parameters chosen, while consistent with the labor literature, are largely ad hoc in a managed money setting as fund entry and exit data is sparse. Our baseline is to start with identical search parameters for both activists and quants.

5.1 Baseline Parameter Results

Figure 1 illustrates the stationary distribution of activists and quants under the Table 1 parameters. The top panels summarize the joint density, while the bottom panels summarize the marginal densities. In equilibrium, a non-degenerate distribution exists where the most likely number of activists is 9 and the most likely number of quants is 7 out of 25 firms in total. This asymmetry is driven by quants needing activist-identified firms to exist leading to lower numbers of quants on average even though both have identical search costs. In separate experiments, the stationary distribution for activists and quants is non-degenerate across a wide range of parameters. Given fees are endogenous, it is optimal for activists and quants to set fees such that they remain in the fund management sector.
While Figure 1 conveys information about the number of activists and quants in equilibrium, Figures 2, 3, and 4 demonstrate how fund market competition impacts assets under management (AUM) through the household’s portfolio problem. Figure 2 plots AUMs as fraction of wealth across the three fund types as a function of fund market competition as summarized by the number of activists \((n)\) and quants \((m)\). Figures 3 and 4 provide different slices of Figure 2 conditional on 1, 5, or 10 quants or activists respectively. These two slices along the quant and activist dimensions are symmetric as the households are indifferent between holding similarly sized activist and quant sectors as both sectors provide the same expected returns at the same fee.

Figure 3 is a good place to understand how competition among funds has diversification and spillover effects. As the number of activist funds in the market increases from, for example, a single fund, the household will initially invest more in the activist sector overall, because it becomes better diversified. But more managed funds also implies more efficient firms in the passive index, a spillover effect that eventually causes AUM among activists to decline with the entry of additional activists. Since quant funds also produce the spillover effect, the point at which AUM begins to decline with additional activists comes sooner when there are many quants in the market. Passive investment is also high when both the number of activists and quants are low. Given activist and quant funds match on single non-overlapping firms, these funds are not well-diversified when small, so the household has less incentive to invest in them relative to the passive fund even though it largely delivers inefficient returns.

Equilibrium fees are summarized in Figures 5 and 6. The top panel of Figure 6 plots the fee rate, common for both activists and quants, for different amounts of fund competition. Equilibrium fees range between slightly above 0 basis points and slightly below 200 basis points. Increased competition, either through more activists or quants, lowers the fee rate. But, the more pertinent measure of fees is not the fee rate. Instead, the present value of fees
collected over a fund’s lifetime is presented as a density for activists and quants in Figure 5 and conditional on the current amount of fund competition in the lower two panels of Figure 6.

The present value of fees in Figure 5 highlights a big difference between activist and quant dynamics. While the present value of fees per activist fund are decreasing with the number of activists (left panel), the same is not true for quants (right panel). On a per-fund basis, the value of quant fees is relatively flat in the number of quants, rising slightly as the number of quants increases. The slight increase reflects a deterrent effect from a market dominated by quants: new funds would attract low AUM, hence fewer managers search for new opportunities. A high number of quants also implies strong spillovers in efficiency to the index, which leads to relatively flat fee present value overall. What explains the activist fee present value generally declining in the number of activists? While a large number of incumbent activists may deter new activists from entering, it does not deter new quants, who have a higher chance of matching with an efficient firm with many activists in the market. Since both this effect and the spillover effect operate in the same direction, activist fee present values are declining in \( n \).

Figure 7 analyzes how household welfare is driven by competition in the fund management sector. Analogous to the early plots, household welfare is represented for differing levels of activists and quants. Even after fees, all things equal, more activists and quants are preferred to less from a welfare perspective as the market for firms is more productive after fees. The plots for some activist, quant combinations are truncated due to that combination having zero probability in the steady state distribution. For example, for \( m = 10 \) quants, the maximal number of activists that still has a positive probability is \( n = 13 \).

However, from the left panel, the households would prefer a high number of activists and a low number of quants in equilibrium from a welfare perspective. Given activists actively find the inefficient firms and quants free ride, a high number of activists is desirable as they
make the market for firms more efficient including the passive fund. Given free entry by
quants though, the second best is to have a setting where activists and quants jointly make
the market more efficient.\footnote{In a current extension, we are exploring settings where the quant can only imperfectly deliver the
efficiency gains of the activist at a lower fee. Here the tradeoff between efficiency and lower fees could
impact the welfare gains to households.}

5.2 Comparative Statics

We also consider a series of comparative statics to better understand the key factors driving
the dynamics of the fund management sector and household choice in equilibrium.

5.2.1 Quant Search Costs

Figure 8 summarizes a comparative static exercise where the quant’s search cost $Q$ is varied
around the baseline parameter choice of $Q = 0.001$. Instead of presenting the stationary
distributions, we plot mean equilibrium quantities to more conveniently summarize the im-
pact of a change in the quant’s search cost. From the two top left plots, the mean number
of activists and mean activist AUM are increasing in the quant’s search cost. As a quant’s
search cost increases, the probability drops that an activist will be picked off implying more
activists at higher quant search costs.

The mean number of quants and mean quant AUM, in the top right panels, are monotonic
though. Starting from a high enough quant search cost, the mean number of quants increases
as search costs fall. Given more quants, activists are picked off at a higher rate. Eventually,
this is detrimental to the number of quants. As search costs continue to fall, this has such a
detrimental impact on the supply of activists that the mean number of quants also fall along
with the mean quant AUM.

Interestingly, the mean fee rate for activists and quants is decreasing in the quant’s
search cost as seen in the bottom left panel. As a quant’s search cost falls, more activists
are driven from the market implying less competition. The remaining activists are then able to charge higher fees. However, this does not translate into a higher present value of fees to the surviving activists as seen in the panel above. This perverse impact on the activist sector as quant search costs fall seriously impacts household welfare due to the bulk of the firms now being inefficient as can be seen in the bottom right panel.

5.2.2 Higher Passive Investing Costs

In our baseline parameters, the cost of investing in the passive index $\pi$ is zero. One way to impact the overall costs of intermediation in the fund management sector is to raise the cost of investing in the index which is then internalized in the optimal fee set by the activists and quants as can be seen in equation (37). Figure 9 summarizes comparative statics for mean equilibrium outcomes as the index fee rises from 0 to 150 basis points.

As expected, the mean fee rate and the present value of fees for activists and quants increase as the substitute asset, the passive index, becomes more expensive as can be seen in the bottom panels. Surprisingly, increasing index fees has an opposite effect on the mean number of activists and quants. As the index fee increases, the mean number of activists falls while the mean number of quants increases. This is driven by a reconfiguration of the steady state between activists and quants. From a competition stand point, higher index fees mean the quants have less competition leading to more quants in equilibrium. This in turn then lessens the desire for more activists to enter the fund management space leading to fewer activists and a non-monotonic activist AUM as a function of index fees.

From a welfare perspective, households are worse off when index fees are higher. First, higher index fees simply make investing in the index more expensive. Second, it also makes the firm market more inefficient given higher index fees crowd out activists who make the market more efficient.
5.2.3 Lower Household Risk Aversion

Figure 10 summarizes a comparative static exercise where the household’s risk aversion $\gamma$ is varied from 2.5 to 4.0. Given $\gamma = 4.0$ was our baseline risk aversion, this comparative static is meant to demonstrate how the desire for less diversification and more expected returns through lower risk aversion impacts the fund management sector. When risk aversion is lower, the mean number of activists falls while the mean number of quants rises. AUMs for both sectors increase though risk aversion falls. With the drop in risk aversion, households are willing to hold less diversified activist positions.

6 Conclusion

We study a general equilibrium production economy with a financial sector. The financial sector contains an actively-managed sector and a passively-managed sector. The actively-managed sector is socially desirable as it helps improve productive efficiency by monitoring firm managers; i.e., through the use of “voice” via activists. The actively-managed sector also contains quant funds, who displace activists, but still hold efficient firms. A representative household allocates its wealth to both the active and passive fund sectors, and we examine a steady state equilibrium where actively and passively managed sectors coexist in positive net supply. Given the model structure, we conduct a series of experiments to characterize the impact on the steady state equilibrium of changes in the matching function between active managers and firms and changes in the costs of intermediation. Our results highlight the role of general equilibrium in determining fund dynamics and fees in the financial sector.
References


<table>
<thead>
<tr>
<th>Parameter</th>
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<tr>
<td>Efficient deterministic cap. depreciation</td>
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<tr>
<td>Manager reservation utility coeff.</td>
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**Table 1: Parameter values.** The table reports the baseline parameter values used in our numerical examples.
Figure 1: Stationary Distribution of Activists and Quants: Base Parameters. The top left figure shows the joint stationary density of the number of quant \((m)\) and activist \((n)\) funds in the market, under example parameter values in Table 1. The top right figure represents the same information as a contour plot. The bottom figures show the marginal stationary densities of the number of activist \((n)\) and quant \((m)\) funds in the market.

Figure 5: Stationary Density and Fees: Base Parameters. In the top row, the figure shows the marginal stationary density of the number of quant \((m)\) and activist \((n)\) funds in the market, under example parameter values in Table 1. The bottom row illustrates the present value of fees, per fund, conditional on the number of funds of that type in the market. Multiple values along the y-axis reflect potential levels of competition from the other fund type, and the size of each circle indicates the likelihood (conditional density) of being in each state of competition.
Figure 2: AUM: Base Parameters. The figure shows assets under management (AUM) in each of the three fund sectors, as a fraction of aggregate wealth, conditional on the state of competition in the fund manager market. Fund market competition is summarized by the tuple \((m, n)\), where \(m\) is the number of quant funds, and \(n\) the number of activist funds. The top panel summarizes total activist AUM, the middle panel summarizes total quant AUM, and the bottom panel summarizes passive AUM. Parameter values are per Table 1.
Figure 3: AUM Conditional on Quants: Base Parameters. The figure shows assets under management (AUM) as a fraction of aggregate wealth in each of the three fund sectors as a function of the number of activists ($n$). The information here summarizes three different quant levels from Figure 2. The top panel summarizes total activist AUM, the middle panel summarizes total quant AUM, and the bottom panel summarizes passive AUM. Parameter values are per Table 1.
Figure 4: AUM Conditional on Activists: Base Parameters. The figure shows assets under management (AUM) as a fraction of aggregate wealth in each of the three fund sectors as a function of the number of quants ($m$). The information here summarizes three different activist levels from Figure 2. The top panel summarizes total activist AUM, the middle panel summarizes total quant AUM, and the bottom panel summarizes passive AUM. Parameter values are per Table 1.
Figure 5: Present Value of Fees Densities: Base Parameters. The figure illustrates the present value of fees, per fund, conditional on the number of funds of that type in the market. Multiple values along the y-axis reflect potential levels of competition from the other fund type, and the size of each circle indicates the likelihood (conditional density) of being in each state of competition. Parameter values are per Table 1.
Figure 6: Fees: Base Parameters. The figure shows properties of fees conditional on the state of competition in the fund manager market. Fund market competition is summarized by the tuple \((m, n)\), where \(n\) is the number of activist funds, and \(m\) the number of quant funds. The top panels summarize the fee rate for both activists and quants, the middle panels summarize the present value of activist fees per fund, and the bottom panels summarize the present value of quant fees per fund. The left panels plot relative to the number of activists \(n\), while the right panels plot relative to the number of quants \(m\). Parameter values are per Table 1.
Figure 7: Household Welfare: Base Parameters. The figure shows properties of household welfare conditional on the state of competition in the fund manager market. Fund market competition is summarized by the tuple \((m, n)\), where \(n\) is the number of activist funds, and \(m\) the number of quant funds. The left panel plots relative to the number of activists \(n\), while the right panel plots relative to the number of quants \(m\). Parameter values are per Table 1.
Figure 8: Vary Quant Search Costs. The effect of changing the quant’s search cost is illustrated in the above figure. With the exception of quant search costs, given on the x-axis, all parameters are per Table 1. The quant search cost parameter $\zeta_Q$ is varied from 0.0001 to 0.002.
Figure 9: Vary Index Fee. The effect of increasing the passive index fee is illustrated in the above figure. With the exception of passive index fee $\tilde{\pi}$, given on the x-axis, all parameters are per Table 1. The passive index fee $\tilde{\pi}$ is varied from 0.00 to 0.015.
Figure 10: Vary Household Risk Aversion. The effect of changing household risk aversion $\gamma$ is illustrated in the above figure. With the exception of household risk aversion, given on the x-axis, all parameters are per Table 1. The household risk aversion $\gamma$ is varied from 2.5 to 4.0.