

# Liquidity Creation as Volatility Risk

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# Liquidity and Volatility

1. Liquidity creation is a key service provided by the financial sector
  - makes it cheaper to pledge income streams from assets
  - enables risk sharing and the allocation of capital
  - underlies banking and market making
2. Volatility is a central feature of financial markets
  - fluctuates widely over time
  - large and time-varying premium for hedging volatility shocks (variance premium)
3. Volatility and liquidity appear to move together
  - financial crises: volatility spikes and liquidity evaporates (Brunnermeier, 2009)
  - Nagel (2012): higher VIX predicts higher returns to liquidity creation (stock reversals)

# This paper

We show theoretically and empirically that:

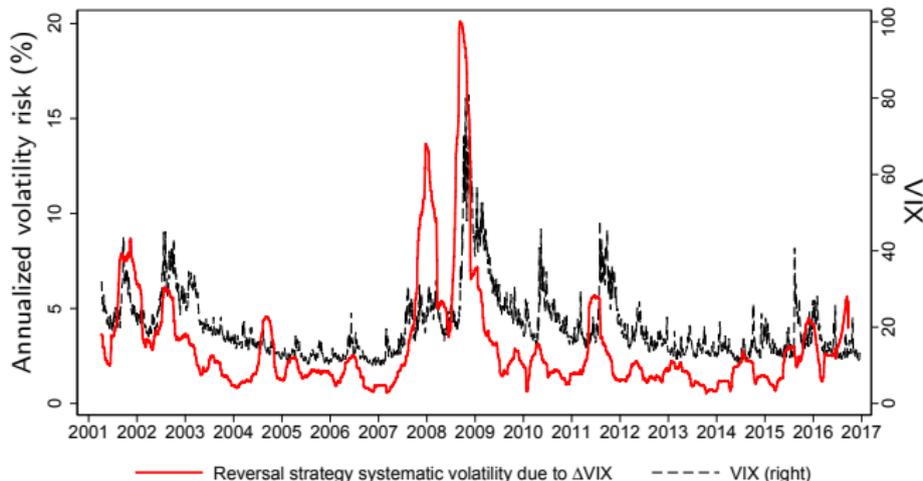
1. Liquidity creation has a built-in negative exposure to volatility shocks
    - when volatility rises  $\rightarrow$  liquidity providers lose
    - fundamental, due only to information asymmetry
    - no financial frictions, liquidity providers are fully diversified
  2. Returns to liquidity creation reflect compensation for its volatility risk exposure
    - show this using stock reversals
    - reversal portfolios have large negative volatility  $\beta$ 's
    - expected reversal return = volatility  $\beta \times$  variance premium
    - when volatility risk increases  $\rightarrow$  expected reversal return rises
- $\Rightarrow$  A new, asset-pricing perspective on liquidity creation and financial intermediation more broadly

# Intuition

1. Liquidity providers absorb order flow from liquidity traders and informed traders
  - problem: don't know how much info out there
  - protect themselves by making prices respond to order flow
  - higher expected vol, more info → prices more sensitive
2. Risk: tomorrow vol turns out to be higher than was expected
  - yesterday, traded at prices that didn't respond enough to order flow
  - prices tomorrow will move further in the direction of yesterday's order flow if vol spikes
  - liquidity providers will lose on all positions (longs fall, shorts rise)
3. Volatility is highly correlated across assets, and with market volatility
  - ⇒ liquidity providers' volatility exposure is undiversifiable, systematic
4. Systematic volatility risk carries a big premium (variance premium)
  - liquidity providers charge for their volatility risk exposure
  - ⇒ when VIX is higher, higher vol risk, higher variance premium
  - liquidity provision strategies earn a higher premium

# Volatility risk exposure of stock reversals

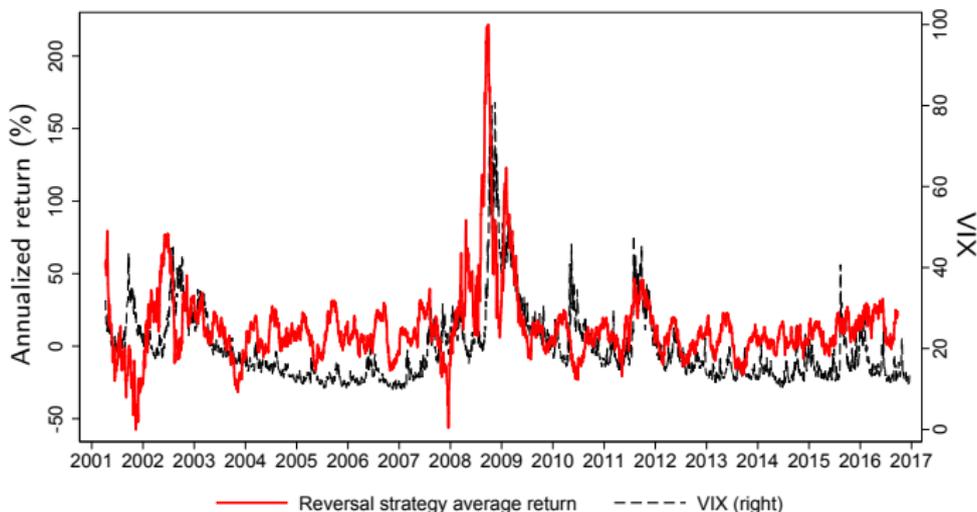
1. Use stock reversals as proxy for returns to liquidity creation
  - sort large-cap stocks into deciles by day's normalized return
  - buy lowest-return decile, sell highest-return decile, hold for five days
2. Reversal strategy's daily  $\beta_{\Delta VIX} = -19$  bps per 1 point  $\Delta VIX$ 
  - big relative to 27 bps average five-day return
  - plot rolling estimate of volatility risk:  $\sigma(\beta_{\Delta VIX} \times \Delta VIX)$



- ⇒ Volatility risk of the strategy is large and time-varying
- ⇒ When VIX is higher → reversal strategy has higher volatility risk

# Volatility risk and the average return of stock reversals

1. Look at the rolling future reversal return against VIX (as Nagel 2012)



2. Reversal strategy return is strongly increasing in VIX  
⇒ Higher VIX → reversal has higher volatility risk → higher return
3. Exposure to volatility risk explains the returns to liquidity creation

# Model

1. Kyle (1985) framework with stochastic volatility
  - three time periods: 0,  $t \in (0, 1)$ , and 1
  - three agents: informed trader, liquidity-demanders, liquidity providers
  - $N$  assets: traded at time 0, pay off at time 1:

$$p_{i,1} = \bar{v}_i + \sigma_{i,1} v_i$$

- $v_i \sim N(0, 1)$  is idiosyncratic; informed knows  $v_i$  at  $t = 0$
  - information more valuable when volatility is higher ( $\sigma_{i,1}$  higher)
  - $\sigma_{i,1}$  is uncertain to everyone, realized at time 1.
2. Time  $t$ : public news arrives about  $\sigma_{i,1}$ 
    - prices respond immediately (but no trading since public news)
    - volatility news not diversifiable, commonality in idio vol

$$\sigma_{i,1} = k_{i,m} \sigma_{m,1} + \varepsilon_{\sigma_i}$$

- $k_{i,m} > 0$  is loading on market vol  $\sigma_{m,1}$
3. Liquidity-demanders: demand  $z_i \sim N(0, \sigma_{z_i}^2)$
  4. Informed trader: demands  $y_i$  to maximize expected time-1 profit

$$\max_{y_i} E_0^Q [y_i (p_{i,1} - p_{i,0}) | v_i]$$

- values profits under economy's risk-neutral measure  $Q$

## Equilibrium pricing

1. Liquidity providers absorb order flow  $X_i = y_i + z_i \rightarrow$  hold  $-X_i$  shares.  
Set  $p_{i,t}$  to break even under  $Q$  measure  $\Rightarrow$  no financial frictions

$$p_{i,0} = E_0^Q [p_{i,1} | X_i] = \bar{v}_i + X_i \frac{E_0^Q [\sigma_{i,1}]}{2\sigma_{z_i}}$$

- $p_{i,0}$  moves in direction of order flow  $X_i$
  - higher  $E_0^Q [\sigma_{i,1}] \rightarrow$  informed has more info  $\rightarrow p_{i,0}$  more sensitive to  $X_i$
2. Let  $\Delta p_{i,0} = (p_{i,0} - \bar{v}_i)$  denote the time-0 price change. Then:

$$-X_i = -\Delta p_{i,0} \frac{2\sigma_{z_i}}{E_0^Q [\sigma_{i,1}]}$$

- $\Rightarrow$  liquidity providers hold a **portfolio of reversals**: they buy assets that go down and short assets that go up, in proportion to  $-\Delta p_{i,0}$
- use reversals to proxy for liquidity provision as in Nagel (2012)

# Volatility risk exposure

1. **Public volatility news** arrives at  $t$ , time-0 order flow more informed than was priced in, liq providers incorporate this into prices

$$\Delta p_{i,t} = E_t^Q[p_{i,1}] - E_0^Q[p_{i,1}] = \frac{X_i}{2\sigma_{z_i}} \left( E_t^Q[\sigma_{i,1}] - E_0^Q[\sigma_{i,1}] \right)$$

- ⇒ prices adjust further in direction of time-0 order flows
- ⇒ longs go down and shorts go up, reversals lose on both sides

2. Market vol betas  $\beta_{i,\sigma_m}$  of liquidity providers' holdings  $-X_i\Delta p_{i,t}$  are:

$$\beta_{i,\sigma_m} = -2\sigma_{z_i} \left( \frac{\Delta p_{i,0}}{E_0^Q[\sigma_{i,1}]} \right)^2 k_{i,m} < 0$$

- $\beta_{i,\sigma_m}$  negative, magnitude increasing in (normalized) return

3. Liquidity providers bear *undiversifiable* volatility risk even though assets' time-1 payoffs are **totally idiosyncratic**

- correlation in second moments induces undiversifiable risk
- contrasts with inventory models, where idio vol is only priced because liquidity providers can't diversify (Stoll 1978, Nagel 2012)

# Predictions summary

1. **Exposure:** Reversals have negative exposure to market vol

$$\beta_{\sigma_m} = \left( \sum_{i=1}^N \beta_{i,\sigma_m} \right) < 0$$

2. **Risk premium:** Reversals earn a large, positive risk premium (liquidity premium) from time 0 to 1

$$E_0^P \left[ \sum_{i=1}^N -X_i \Delta p_{i,1} \right] = \beta_{\sigma_m} \underbrace{\left( E_0^P [\sigma_{m,1}] - E_0^Q [\sigma_{m,1}] \right)}_{\text{variance premium} \ll 0} > 0$$

- large variance premium in the data: VIX  $\gg$  realized vol of S&P 500

3. **Time-series predictability:**

- higher VIX, higher vol risk, VRP  $\rightarrow$  **higher future reversal returns**

4. **Cross-section predictability:**

① higher  $|\Delta p_{i,0}| \rightarrow$  **more negative**  $\beta_{\sigma_m}$

② more negative  $\beta_{\sigma_m} \rightarrow$  **larger avg. reversal return**

# Data and empirical strategy

1. Construct reversal portfolios empirically to measure risk and returns to liquidity provision
  - ① Each day, sort stocks into quintiles by size and then deciles by return normalized by rolling standard deviation and weighted by dollar volume
  - ② focus on period since decimilization: 4/9/2001 to 12/31/2016 (3,958 days), when liquidity provision became competitive
  - ③ drop penny stocks and earnings announcements (public news events)
  - ④ hold portfolios for one to five days as in Nagel, 2012 → *not* HFT
2. Look at the entire cross section of reversals:
  - buy low-return deciles, sell high-return deciles:  
1-10 ("Lo-Hi"), 2-9, . . . , 5-6
  - portfolios capture decreasing amounts of liquidity provision

## Average returns and CAPM alphas

$$R_{t,t+5}^P = \alpha^P + \sum_{s=1}^5 \beta_s^P R_{t+s}^M + \epsilon_{t,t+5}^P$$

	Lo-Hi	5-day average return (%)			
		2-9	3-8	4-7	5-6
Small	1.16	0.56	0.21	0.05	0.04
2	0.65	0.30	0.17	0.03	-0.03
3	0.35	0.24	0.01	0.11	-0.01
4	0.22	0.23	0.13	0.06	0.01
<b>Big</b>	<b>0.27</b>	<b>0.25</b>	<b>0.18</b>	<b>0.11</b>	<b>0.05</b>

	Lo-Hi	5-day CAPM alpha (%)			
		2-9	3-8	4-7	5-6
Small	1.14	0.55	0.20	0.04	0.04
2	0.62	0.30	0.16	0.02	-0.03
3	0.34	0.23	-0.00	0.10	-0.01
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<b>Big</b>	<b>0.25</b>	<b>0.24</b>	<b>0.18</b>	<b>0.11</b>	<b>0.05</b>

- Large-cap reversal average five-day return: 27 bps (13.6% annual), Sharpe ratio 0.6
  - small-stock reversals are larger, portfolios 1 and 2 < 0.5% mkt cap
  - CAPM alphas  $\approx$  average returns  $\Rightarrow$  CAPM cannot price the reversals
- Avg returns increase in amount of liquidity provision (5-6 to Hi-Lo)

## Volatility risk exposure

$$R_{t,t+5}^P = \alpha_p + \sum_{s=1}^5 \beta_s^{P,VIX} \Delta VIX_{t+s} + \epsilon_{t,t+5}^P$$

		5-day $\Delta VIX$ beta			
	Lo-Hi	2-9	3-8	4-7	5-6
Small	-0.81	-0.49	-0.57	-0.26	-0.31
2	-0.82	-0.34	-0.24	-0.31	0.03
3	-0.57	-0.26	-0.36	-0.32	-0.01
4	-0.54	-0.26	-0.18	0.01	-0.04
Big	-0.64	-0.34	-0.09	-0.01	-0.01

		5-day $\Delta VIX$ beta t-statistic			
	Lo-Hi	2-9	3-8	4-7	5-6
Small	-2.66	-3.29	-3.02	-1.31	-1.78
2	-3.47	-2.21	-1.82	-2.39	0.25
3	-3.63	-2.52	-3.82	-4.16	-0.09
4	-4.30	-2.78	-2.70	0.13	-0.93
Big	-4.28	-3.09	-1.34	-0.25	-0.31

- Reversal strategy has a large negative beta to  $\Delta VIX$ 
  - large-cap reversal drops by 64 bps per 5-point VIX increase (1.3 standard deviations); big relative to average return (27 bps)
- Beta magnitude increasing in amount of liquidity provision

# Predictability regressions

- Model: higher VIX  $\rightarrow$  higher VRP  $\rightarrow$  higher reversal return
  - predictability coefficient increasing in amount of liquidity provision

$$R_{t,t+5}^P = \alpha^P + \beta^P VIX_t + \epsilon_{t,t+5}^P$$

		5-day VIX loading ( $\times 10^2$ )				
		Lo-Hi	2-9	3-8	4-7	5-6
Small		3.53	3.48	3.51	2.72	0.16
2		7.01	3.14	2.68	1.40	-0.27
3		4.84	2.98	1.16	0.93	-0.10
4		2.94	2.33	1.52	-0.04	0.44
Big		5.37	3.69	1.74	0.67	0.08

		5-day $R^2$ (%)				
		Lo-Hi	2-9	3-8	4-7	5-6
Small		0.09	0.19	0.26	0.16	0.00
2		0.95	0.37	0.35	0.11	0.00
3		0.72	0.70	0.15	0.08	0.00
4		0.46	0.71	0.42	0.00	0.05
Big		2.18	2.11	0.77	0.15	0.00

- $VIX$  predicts reversal strategy returns
  - extends result of Nagel (2012) to cross section
  - predictive coefficients increasing in liquidity provision
  - very high  $R^2$  for large stocks given five-day horizon

# Fama-Macbeth regressions

		Factor premia			R.m.s.	p-value
	Market	t-stat.	$\Delta VIX$	t-stat.		
(1)	0.03	2.06			0.18	0.00
(2)	0.05	3.04	-0.49	-8.57	0.14	0.00

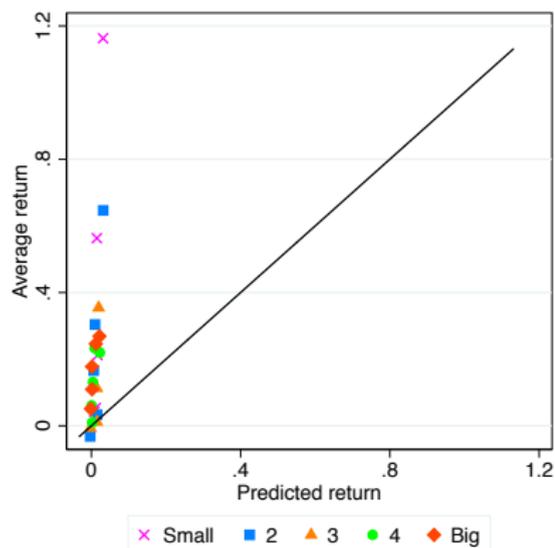
		(1) CAPM pricing error			
	Lo-Hi	2-9	3-8	4-7	5-6
Small	1.13	0.55	0.20	0.04	0.03
2	0.61	0.29	0.16	0.02	-0.03
3	0.33	0.23	-0.00	0.10	-0.00
4	0.20	0.22	0.13	0.06	0.01
Big	0.25	0.23	0.18	0.11	0.05

		(2) Market plus $\Delta VIX$ pricing error			
	Lo-Hi	2-9	3-8	4-7	5-6
Small	0.79	0.28	-0.17	-0.02	-0.17
2	0.32	0.10	0.00	-0.01	-0.06
3	0.06	0.17	-0.12	0.03	-0.07
4	0.04	0.09	-0.02	0.08	0.00
Big	-0.07	0.07	0.10	0.13	-0.00

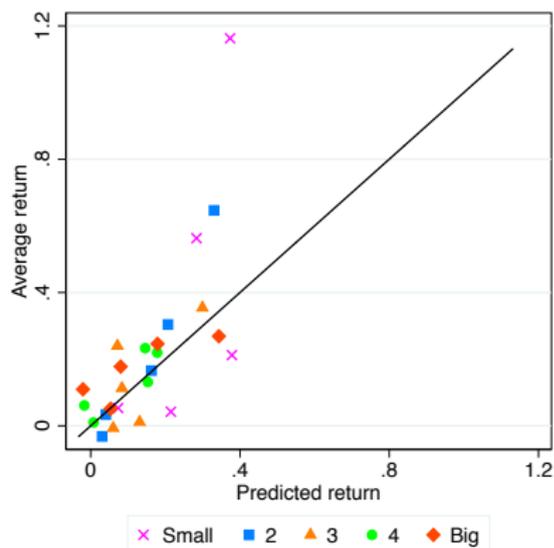
1.  $\Delta VIX$  factor explains the reversal strategy returns of large- and mid-cap stocks. Large and significant premium

# Fama-Macbeth regressions

(1) CAPM



(2) Market plus  $\Delta VIX$



1.  $\Delta VIX$  factor explains the reversal strategy returns of large- and mid-cap stocks. Large and significant premium

# Is the implied price of volatility risk consistent with other markets?

1. Volatility risk is traded directly in option markets
    - VIX is the price of a basket of options that replicates the realized variance of the S&P 500 over next 30 days
    - However,  $\Delta VIX$  is not a return because basket changes daily
  2. We replicate the VIX using S&P 500 options and use the change in the price of a given basket to get a **VIX return**
    - We also use VIXN, the near-term component of VIX ( $\approx 22$  days) to capture better horizon of reversal strategy  $\rightarrow$  **VIXN return**
  3. Average daily **VIX return is  $-1.54\%$** , **VIXN return is  $-2.01\%$** 
    - in line with variance premium literature (e.g. Carr and Wu, 2008; Bollerslev, Tauchen, and Zhou, 2009, Drechsler and Yaron 2010)
- $\Rightarrow$  Implied price of risk:  **$-22$  bps for  $\Delta VIX$**  and  **$-35$  bps for  $\Delta VIXN$**

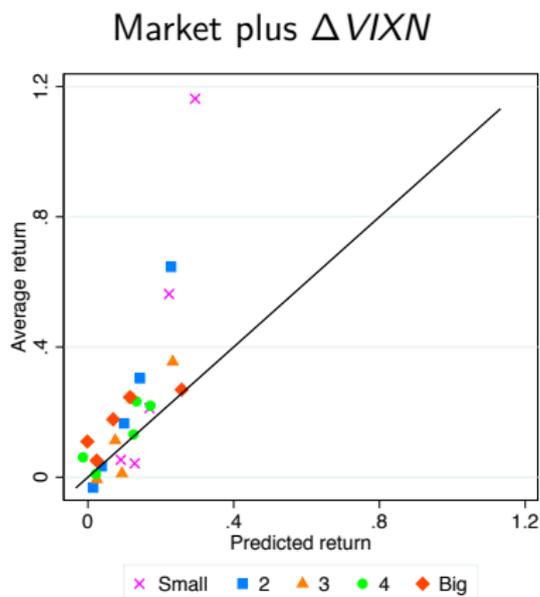
# Pricing regressions with an options-based price of risk

Pricing error using VIX return					
	Lo-Hi	2-9	3-8	4-7	5-6
Small	0.99	0.44	0.04	0.02	-0.05
2	0.50	0.21	0.09	0.01	-0.04
3	0.22	0.21	-0.05	0.07	-0.03
4	0.14	0.17	0.06	0.07	0.01
Big	0.11	0.16	0.14	0.12	0.03

Pricing error using VIXN return					
	Lo-Hi	2-9	3-8	4-7	5-6
Small	0.87	0.34	0.04	-0.04	-0.09
2	0.42	0.16	0.07	-0.00	-0.05
3	0.12	0.12	-0.08	0.04	-0.03
4	0.05	0.10	0.01	0.07	-0.01
Big	0.01	0.13	0.11	0.11	0.03

1. Near-term volatility risk priced the same in reversals and options
  - the options-based price of  $\Delta VIX$  explains most of the reversal return for large stocks (pricing error falls from 25 bps to 11 bps)
  - the near-term  $\Delta VIXN$  fully explains it (pricing error is just 1 bp)
  - there remain unexplained returns in very small stocks, room for market segmentation

# Pricing regressions with an options-based price of risk



1. Options-based price of  $\Delta VIXN$  explains reversal returns of large- and mid-cap stocks
2. Returns to liquidity creation reflect risks priced in financial markets more broadly
  - as opposed to particular financial conditions of the liquidity providers

# Takeaways

1. The connection between volatility and liquidity is fundamental
2. Exposure to asymmetric information  $\Rightarrow$  exposure to volatility risk
3. Large and volatile variance premium  $\Rightarrow$  explains level and variation of liquidity premium
4. Returns to liquidity creation reflect the high premium for volatility risk in financial markets
5. A new, asset-pricing perspective on the risks and returns to financial intermediation

# APPENDIX

## Reversal portfolio summary statistics

		Market cap (billions)			
	Lo-Hi	2-9	3-8	4-7	5-6
Small	0.05	0.05	0.05	0.05	0.05
2	0.16	0.16	0.17	0.17	0.17
3	0.43	0.44	0.44	0.44	0.44
4	1.35	1.37	1.37	1.37	1.37
Big	49.57	54.15	56.02	56.02	55.40

		Amihud illiquidity ( $\times 10^6$ )			
	Lo-Hi	2-9	3-8	4-7	5-6
Small	33.60	21.08	14.42	10.34	8.58
2	5.73	3.98	2.79	2.07	1.70
3	1.36	1.00	0.71	0.52	0.43
4	0.30	0.22	0.16	0.11	0.09
Big	0.03	0.02	0.01	0.01	0.01

1. Large-cap portfolios  $\approx 96.4\%$  of market value
  - liquid, low transaction costs

## Reversal portfolio summary statistics

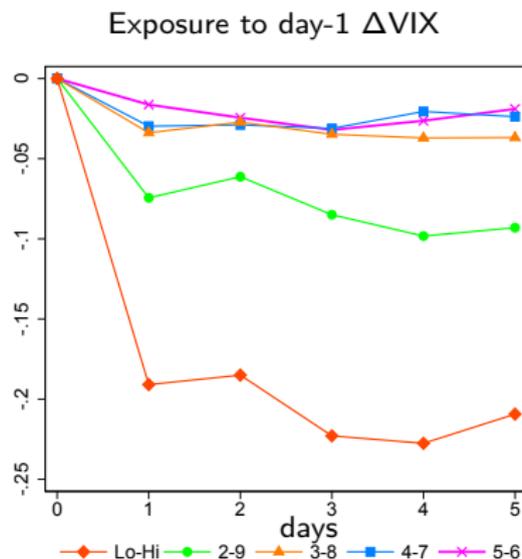
		Sorting-day returns (%)			
	Lo-Hi	2-9	3-8	4-7	5-6
Small	-24.36	-6.92	-4.21	-2.34	-0.74
2	-17.54	-6.05	-3.73	-2.07	-0.64
3	-14.77	-5.43	-3.34	-1.87	-0.60
4	-11.97	-4.70	-2.92	-1.64	-0.52
Big	-7.45	-3.43	-2.13	-1.18	-0.38

		Share turnover (%)			
	Lo-Hi	2-9	3-8	4-7	5-6
Small	10.28	7.37	6.71	6.28	6.07
2	7.84	4.45	3.85	3.60	3.42
3	6.41	3.11	2.63	2.46	2.36
4	5.59	2.76	2.44	2.25	2.21
Big	3.28	2.13	1.99	1.89	1.83

2. Reversal strategy has large negative sorting-day return (by construction)
- larger for small stocks because sorting is by normalized return
  - reversal associated with high share turnover, demand for liquidity

# Volatility risk exposure persistence

1. Model predicts vol shocks have a permanent effect on the reversal strategy:  $\Delta p_{i,t} = \frac{X_i}{2\sigma_{z_i}} \left( E_t^Q [\sigma_{i,1}] - E_0^Q [\sigma_{i,1}] \right)$ 
  - in contrast, inventory models imply this effect also reverses



- Impact of  $\Delta VIX$  shock does **not** reverse

# Average returns and CAPM alphas

$$R_{t,t+5}^P = \alpha^P + \sum_{s=1}^5 \beta_s^P R_{t+s}^M + \epsilon_{t,t+5}^P$$

	5-day average return (%)				
	Lo-Hi	2-9	3-8	4-7	5-6
Small	1.16	0.56	0.21	0.05	0.04
2	0.65	0.30	0.17	0.03	-0.03
3	0.35	0.24	0.01	0.11	-0.01
4	0.22	0.23	0.13	0.06	0.01
Big	0.27	0.25	0.18	0.11	0.05

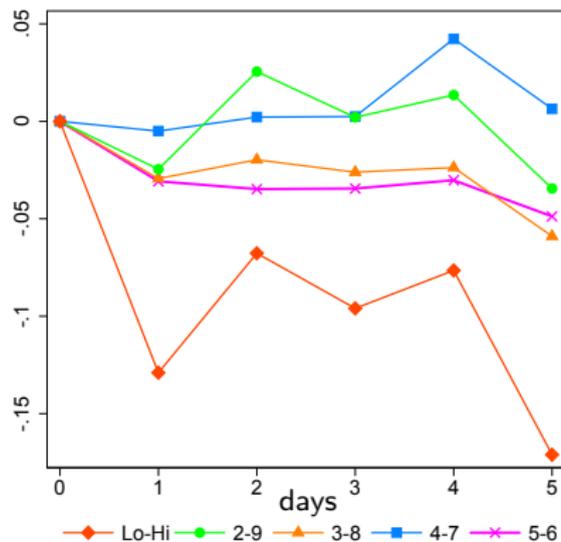
	5-day standard deviation (%)				
	Lo-Hi	2-9	3-8	4-7	5-6
Small	10.54	7.11	6.19	5.99	5.89
2	6.44	4.59	4.03	3.82	3.74
3	5.11	3.18	2.71	3.02	2.28
4	3.88	2.47	2.10	1.92	1.77
Big	3.25	2.28	1.78	1.55	1.31

	5-day CAPM alpha (%)						5-day CAPM alpha t-statistic				
	Lo-Hi	2-9	3-8	4-7	5-6		Lo-Hi	2-9	3-8	4-7	5-6
Small	1.14	0.55	0.20	0.04	0.04	Small	6.57	4.88	1.97	0.44	0.39
2	0.62	0.30	0.16	0.02	-0.03	2	5.83	3.96	2.43	0.33	-0.50
3	0.34	0.23	-0.00	0.10	-0.01	3	3.85	4.31	-0.03	2.25	-0.14
4	0.20	0.23	0.13	0.06	0.01	4	3.13	5.54	3.82	2.00	0.31
Big	0.25	0.24	0.18	0.11	0.05	Big	4.51	6.48	6.24	4.35	2.50

- Large-stock reversal strategy has an average annual return of 13.6% (= 0.27% × 252/5), volatility 23%, Sharpe ratio 0.59
  - small-stock reversal returns are larger but more volatile
  - CAPM alphas ≈ average returns ⇒ CAPM cannot price reversals

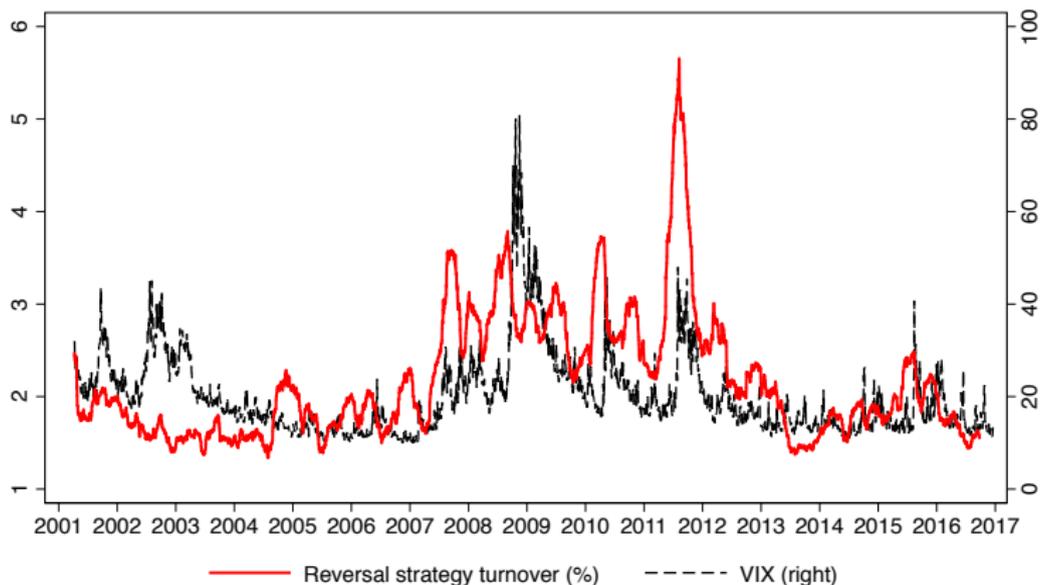
# Volatility risk exposure persistence, controlling for $R^M$

Exposure to day-1  $\Delta VIX$ , controlling for  $R^M$



1. Impact of  $\Delta VIX$  shock is persistent, as predicted by model
  - goes against view that liquidity providers are offloading inventory due to a tightening VaR constraint (impact would be transitory)

# Reversal strategy turnover



## 1. Reversal strategy turnover increasing in VIX

- higher quantity and premium  $\Rightarrow$  shift in liquidity demand curve
- goes against financial constraints theories, which work through shifts in supply curve (e.g., VaR constraint)